LaACES Student Ballooning Course

Errors and Uncertainty Part I

Error In Scientific Measurements

- When we discuss error in a scientific, we are making a statement about how confident we are the true value of a quantity is close to our reported value (our best guess)
- It is quantitative numerical statement
  - I think there's 95% that tree is between 10 and 12 ft. and my best guess is 11 ft.
- Every measured quantity has some uncertainty to it
- It is not incorrect measurement or human error, if you had evidence that a measurement was not correctly done why would you use it

LaACES Student Ballooning Course

Two Types of Error

Random (Statistical)

- Random fluctuation around a true or average value
- Assumed to follow some kind statistical distribution
- Because of that you can make mathematical calculations
- Expect the errors or average out over many measurements
- Examples: electrical noise, mechanical gage reading parallax, radioactive decay, …

Systematic

- NOT Random
- Will not be reduced with increased measurements
- No “correct” mathematical treatment
- Examples: incorrectly calibrated measurement device, limits in measurement resolution

LaACES Student Ballooning Course

Assessing Random Error

- We can measure the random error by looking at the standard deviation, the spread in repeated measurements
- You can’t tell anything from single measurement
- The Standard Deviation tells you how much spread between individual measurements
- The standard deviation of the mean tells you the error when you average several measurements together

- (Sample) **Standard Deviation** (Error in a SINGLE MEASUREMENT)

$$\sigma_s = \sqrt{\frac{\Sigma(x_i - X)^2}{N-1}}$$

- **Standard Deviation of the Mean** (Error of the Average)

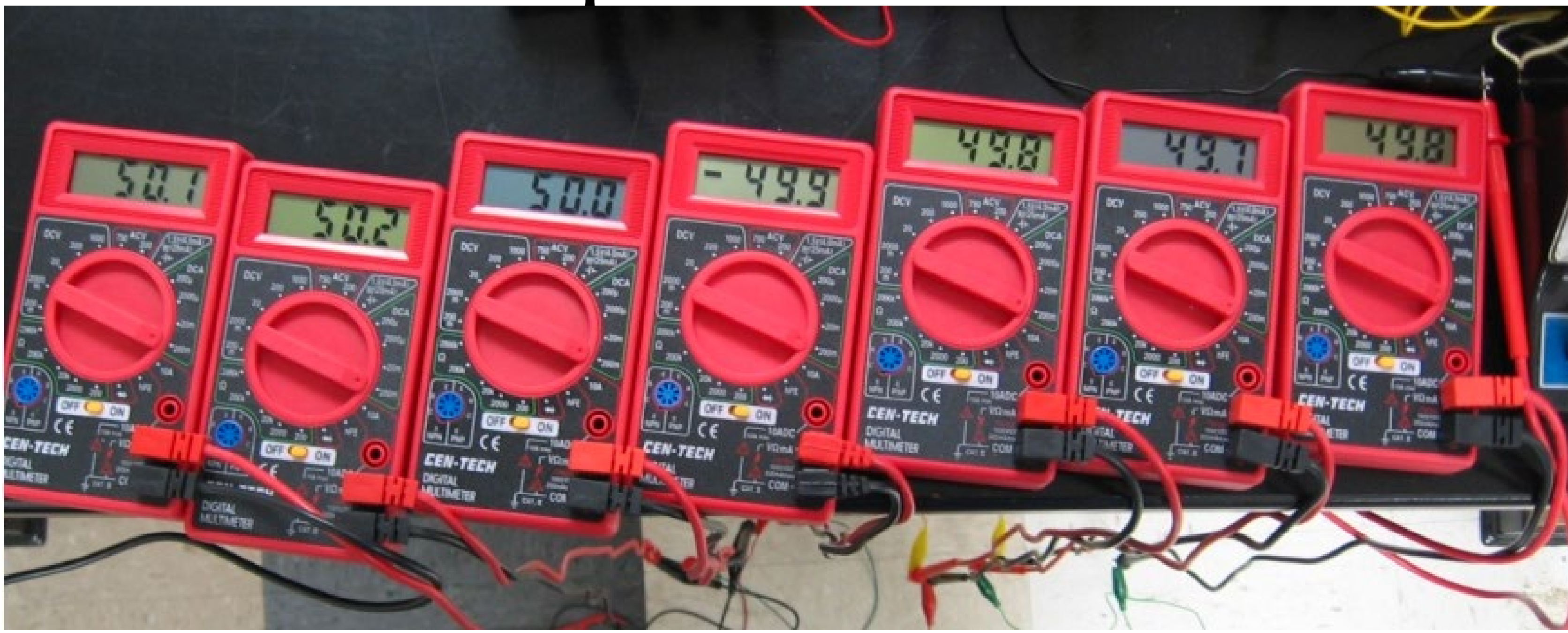
$$\sigma_m = \sqrt{\frac{\sigma_s^2}{N}}$$

Assessing Systematics

- Could compare unrelated measurements or make measurements a very well know object using our measurement system (calibration)
  - Weighing a standardized weight
- Manufacturers will often give a specification of accuracy in the manual for a piece of equipment
  - 1% of Full Scale, 5% of measured value, 2 digits
- The smallest possible increment able to be read with the measurement device
  - Scale markings, digital readout, Analog to Digital Resolution

LaACES Student Ballooning Course

Example Ammeters in series



- Several Multimeters placed in series to measure the same current
- 50.1, 50.2, 50.0, 49.9, 49.8, 49.7, 49.8 mA

Ammeters continued

- Source of differences
- DMM scatter is about 0.2 mA (50 ± 0.2 mA)
  - Manufacturer accuracy: 1% of full scale (2 mA)
  - Each individual DMM repeatable to 0.1 mA
  - The meter are consistent with each other within the specified systematic, but repeated measurement with the same meter will show same variation relative to others
    - One meter will consistently read 0.2 mA compared to another
  - The systematic error is large compared to the random error
    - Taking the standard deviation will ~0 mA because the random error is smaller than the scale of the meter

Other Sources of Systematic Error

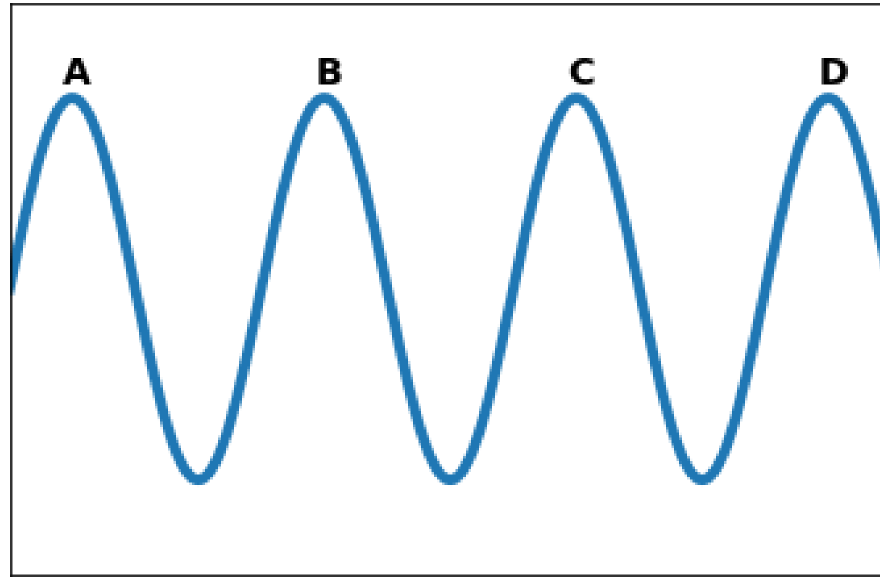
- Often the experimental setup itself can be a source of Systematic Error
  - When measuring a water bath temperature where was the thermometer relative to heater/temperature probe?
    - Letting the thermometer touch the glass will cause it be hotter than the SkeeterSat probe
  - Was quantity measured changing? (maybe heating or cooling)
    - If always read thermometer first
  - Fast enough to have a change larger than your error?

Minimizing Systematic Error

- Since the systematic error limits the accuracy in our measurement minimize it in our experimental setup before we begin to take measurement
- Use the smallest possible scale or highest resolution to measure the value
  - Measuring a 1.5 V battery with the 2V scale rather than 20V
- Let the value stabilize if using a "slow" measurement or slow down the speed of the change
  - Water heats/cools slower than air, a large water bath slower than a small one
- Try take the measurement in a consistent and unbiased way
  - High precision pressure gages often have a mirror behind the needle, if you are looking at the needle directly straight on the needle will block its own reflection

Independent Measurements

- One important precaution is to ensure that all your repeated measurements are independent
- Independent means that the value of one measurement is not affected by any of the other measurements
- This is done to ensure that the random errors of the measurements do of each measurement have nothing to do with each other



For example. we can measure the wavelength above by measuring the peak to peak distance above. We can measure A to B, however then the distance B to C also depends on B so it is not independent of our first measurement. The distance from C to D is.

LaACES Student Ballooning Course

Fitting and Collecting Enough Data

- We want to compare our measurements with a model (A mathematical representation of the data like a line or exponential curve)
- Want to ensure that there is enough points to tell if the model is good representation (averaged measurements should be considered a single point)
- In general for you can you can always perfectly fit a curve that as the same number of points as there are constants in its equation
  - Example a line has 2 constants, slope and y-intercept, and you can always connect a line between 2 points
  - Because you can just perfectly fit with any arbitrary points this doesn't tell you anything meaningful about the data
  - Collect many more data points than constants you plan to use for fitting
- The shape of the curve can be hard to determine if the point are too close together
  - Zooming in on a curve it will look like a line
  - If the point are closer than the error you will just be fitting the random errors
  - Maximize the total range of measurements you can make, sometimes the behavior changes over different ranges

How many datapoints to take

- More data is always better, right? Not always
  - More data takes more time to collect and more time to process
- Can be limited by systematics
  - Since systematic errors can not be averaged repeated measurements once σ_m is smaller than the systematic error, more measurements will not decrease your total error
  - You can not measure a metal bar in nm with a ruler marked in mm no matter how many measurements you repeat
- What range of data do you want the fit to be valid for
  - Interpolation (estimating between measured data points) is better than extrapolation (estimating below or above the range of measured values)
  - Sensor behavior may change over different ranges, uncertainties in fits get larger the farther you are from measured values

Have a plan for successful experiment

- Set up your experiment to minimize your estimated systematics
- Take repeated initial measurement to estimate random error
- Estimate the number of repeated measurements you need to make
- Determine the expected type of fit (linear, quadratic, exponential) and estimate number of data points you need over what range
  - Reviewing data sheets or theory of operation of the sensor
- Record your measurement tool/scale so you can determine systematic errror