Calculating the propagation of normal errors:

The particular equation for propagation of normal errors through a given function is obtained from the following general formula:

$$\sigma_f^2 = \sum \left[ \left( \frac{\partial f}{\partial x_i} \right)^2 \sigma_{x_i}^2 \right]$$

Examples:

1) \( f(x) = Ax \); \( \frac{\partial f}{\partial x} = A \); \( \sigma_f^2 = A^2 \sigma_x^2 \); \( \sigma_f = A \sigma_x \)

2) \( f(x, y) = x - y \); \( \frac{\partial f}{\partial x} = 1 \); \( \frac{\partial f}{\partial y} = -1 \); \( \sigma_f^2 = \sigma_x^2 + \sigma_y^2 \)

3) \( f(x, y) = x + y \); \( \frac{\partial f}{\partial x} = 1 \); \( \frac{\partial f}{\partial y} = 1 \); \( \sigma_f^2 = \sigma_x^2 + \sigma_y^2 \)

4) \( f(x, y) = xy \); \( \frac{\partial f}{\partial x} = y \); \( \frac{\partial f}{\partial y} = x \); \( \sigma_f^2 = y^2 \sigma_x^2 + x^2 \sigma_y^2 \); \( \frac{\sigma_f^2}{f^2} = \frac{y^2 \sigma_x^2}{(xy)^2} + \frac{x^2 \sigma_y^2}{(xy)^2} \)

5) \( f(x, y) = \frac{x}{y} \); \( \frac{\partial f}{\partial x} = \frac{1}{y} \); \( \frac{\partial f}{\partial y} = -\frac{x}{y^2} \); \( \sigma_f^2 = \left( \frac{1}{y} \right)^2 \sigma_x^2 + \left( -\frac{x}{y^2} \right)^2 \sigma_y^2 \)

6) \( f(x_i) = \frac{\sum_{i=1}^{n} x_i}{n} \); \( \frac{\partial f}{\partial x_i} = \frac{1}{n} \); \( \sigma_f^2 = \left( \frac{1}{n} \right)^2 \left( \sum_{i=1}^{n} \sigma_i^2 \right) \)

For high statistics measurements (i.e. a count of \( N = 100 \) or more per individual measurement) the statistical uncertainty of each measurement is generally \( \sigma = \sqrt{N} \) and you would have a measurement of \( N \pm \sigma \)

For low statistics measurements (i.e. counts of \( N < 50 \) or 60 per individual measurement) you will need to use Poisson statistics to get an upper error, \( \sigma_u \), and a lower error, \( \sigma_l \). You would then have a measurement of \( N^{+\sigma_u}_{-\sigma_l} \). Reference: N. Gehrels, “Confidence Limits for Small Numbers of Events in Astrophysics Data”, Astrophysical Journal, 303:336-346, 1986 April 1

When propagating asymmetric error bars you propagate the upper error bars separately from the lower error bars according to the equations above.