Calculating the propagation of normal errors:

The particular equation for propagation of normal errors through a given function is obtained from the following general formula:

$$\sigma_f^2 = \sum_i \left[\left(\frac{\partial f}{\partial x_i}\right)^2 \sigma_{x_i}^2\right]$$

Examples:

1) $f(x) = Ax$; $\frac{\partial f}{\partial x} = A$; $\sigma_f^2 = A^2\,\sigma_x^2$; $\sigma_f = A\sigma_x$

2) $f(x,y) = x - y$; $\frac{\partial f}{\partial x} = 1$; $\frac{\partial f}{\partial y} = -1$; $\sigma_f^2 = \sigma_x^2 + \sigma_y^2$

3) $f(x,y) = x + y$; $\frac{\partial f}{\partial x} = 1$; $\frac{\partial f}{\partial y} = 1$; $\sigma_f^2 = \sigma_x^2 + \sigma_y^2$

4) $f(x,y) = xy$; $\frac{\partial f}{\partial x} = y$; $\frac{\partial f}{\partial y} = x$; $\sigma_f^2 = y^2\sigma_x^2 + x^2\sigma_y^2$; $\frac{\sigma_f^2}{f^2} = \frac{y^2\sigma_x^2}{(xy)^2} + \frac{x^2\sigma_y^2}{(xy)^2}$

$\frac{\sigma_f^2}{f^2} = \frac{\sigma_x^2}{(x)^2} + \frac{\sigma_y^2}{(y)^2}$; $\sigma_f^2 = f^2\left[\frac{\sigma_x^2}{(x)^2} + \frac{\sigma_y^2}{(y)^2}\right]$

5) $f(x,y) = \frac{x}{y}$; $\frac{\partial f}{\partial x} = \frac{1}{y}$; $\frac{\partial f}{\partial y} = -\frac{x}{y^2}$; $\sigma_f^2 = \left(\frac{1}{y}\right)^2\sigma_x^2 + \left(-\frac{x}{y^2}\right)^2\sigma_y^2$;

$\frac{\sigma_f^2}{f^2} = \left(\frac{y}{x}\right)^2\left(\frac{1}{y}\right)^2\sigma_x^2 + \left(\frac{y}{x}\right)^2\left(-\frac{x}{y^2}\right)^2\sigma_y^2$; $\frac{\sigma_f^2}{f^2} = \frac{\sigma_x^2}{(x)^2} + \frac{\sigma_y^2}{(y)^2}$; $\sigma_f^2 = f^2\left[\frac{\sigma_x^2}{(x)^2} + \frac{\sigma_y^2}{(y)^2}\right]$

6) $f(x_i) = \frac{\sum_{i=1}^n x_i}{n}$; $\frac{\partial f}{\partial x_i} = \frac{1}{n}$; $\sigma_f^2 = \left(\frac{1}{n}\right)^2\left(\sum_{i=1}^n \sigma_i^2\right)$

For high statistics measurements (i.e. a count of $N = 100$ or more per individual measurement) the statistical uncertainty of each measurement is generally $\sigma = \sqrt{N}$ and you would have a measurement of $N \pm \sigma$

For low statistics measurements (i.e. counts of $N < 50$ or 60 per individual measurement) you will need to use Poisson statistics to get an upper error, σ_U, and a lower error, σ_L. You would then have a measurement of $N_{-\sigma_L}^{+\sigma_U}$. Reference: N. Gehrels, *"Confidence Limits for Small Numbers of Events in Astrophysics Data",* Astrophysical Journal, **303**:336-346, 1986 April 1

When propagating asymmetric error bars you propagate the upper error bars separately from the lower error bars according to the equations above.