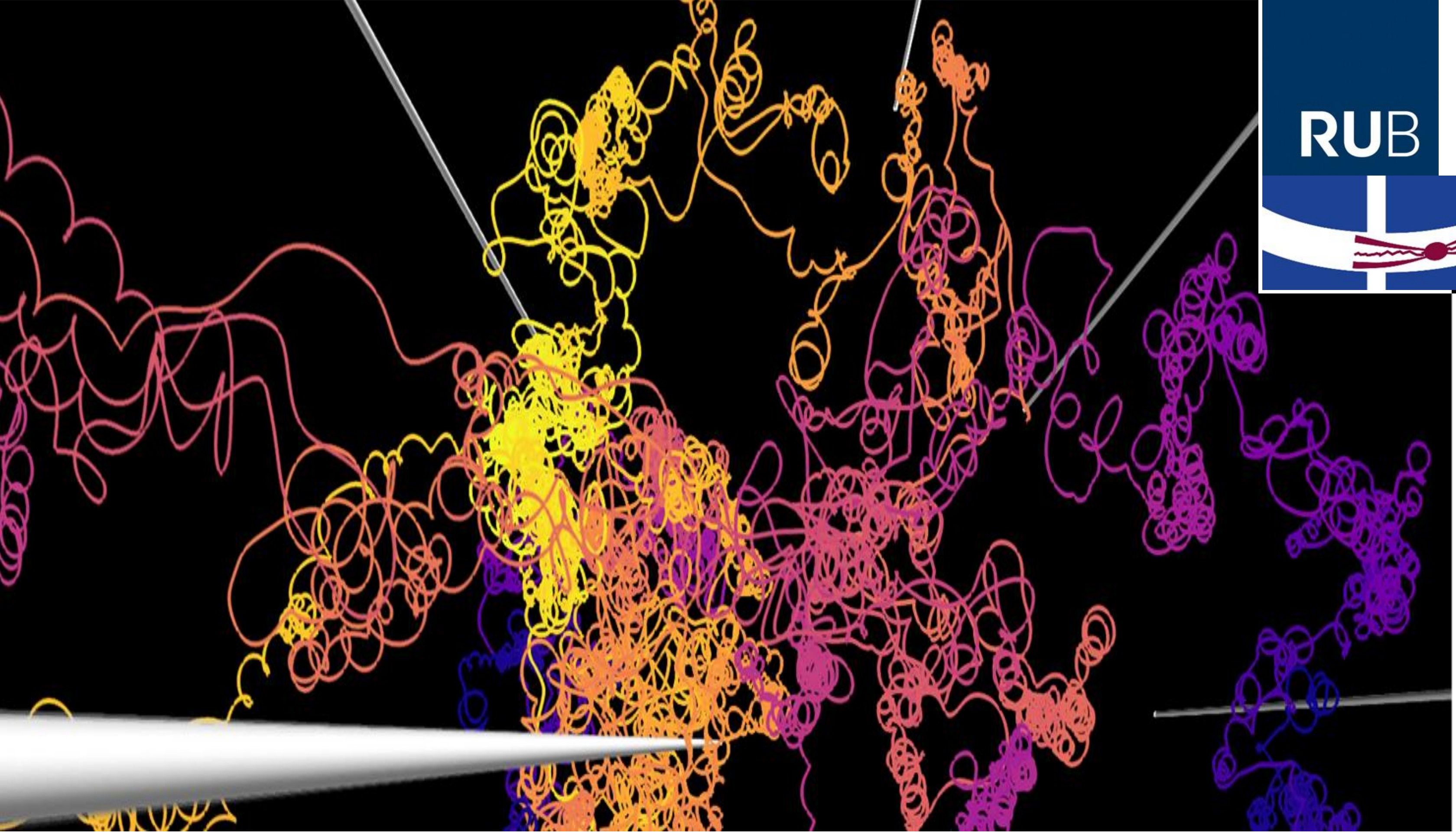


RUHR-UNIVERSITÄT BOCHUM

The transition between Galactic and extra-galactic cosmic rays

Summerschool August 2018 – Erice, Italy

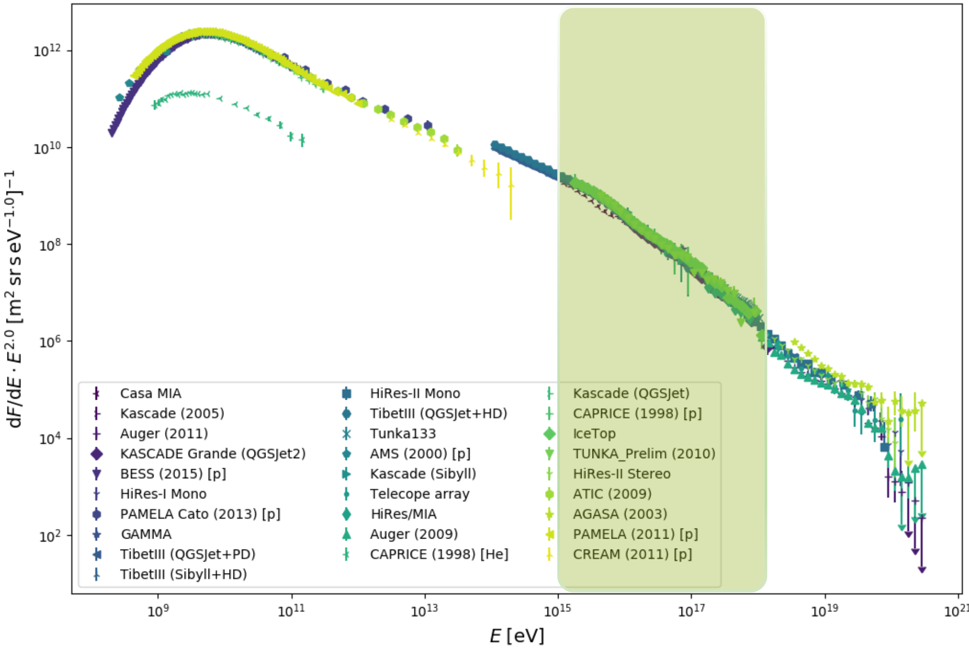
Lukas Merten, Chad Bustard, Ellen Zweibel, Julia Becker Tjus

Where are CRs in the shin region from?

- Cosmic ray can be accelerated at the GTS (Bustard+ 2016/1017)

- Can they diffuse back into the Galaxy?
  - What are the time scales?
  - How are their properties changed?

- What about secondaries?



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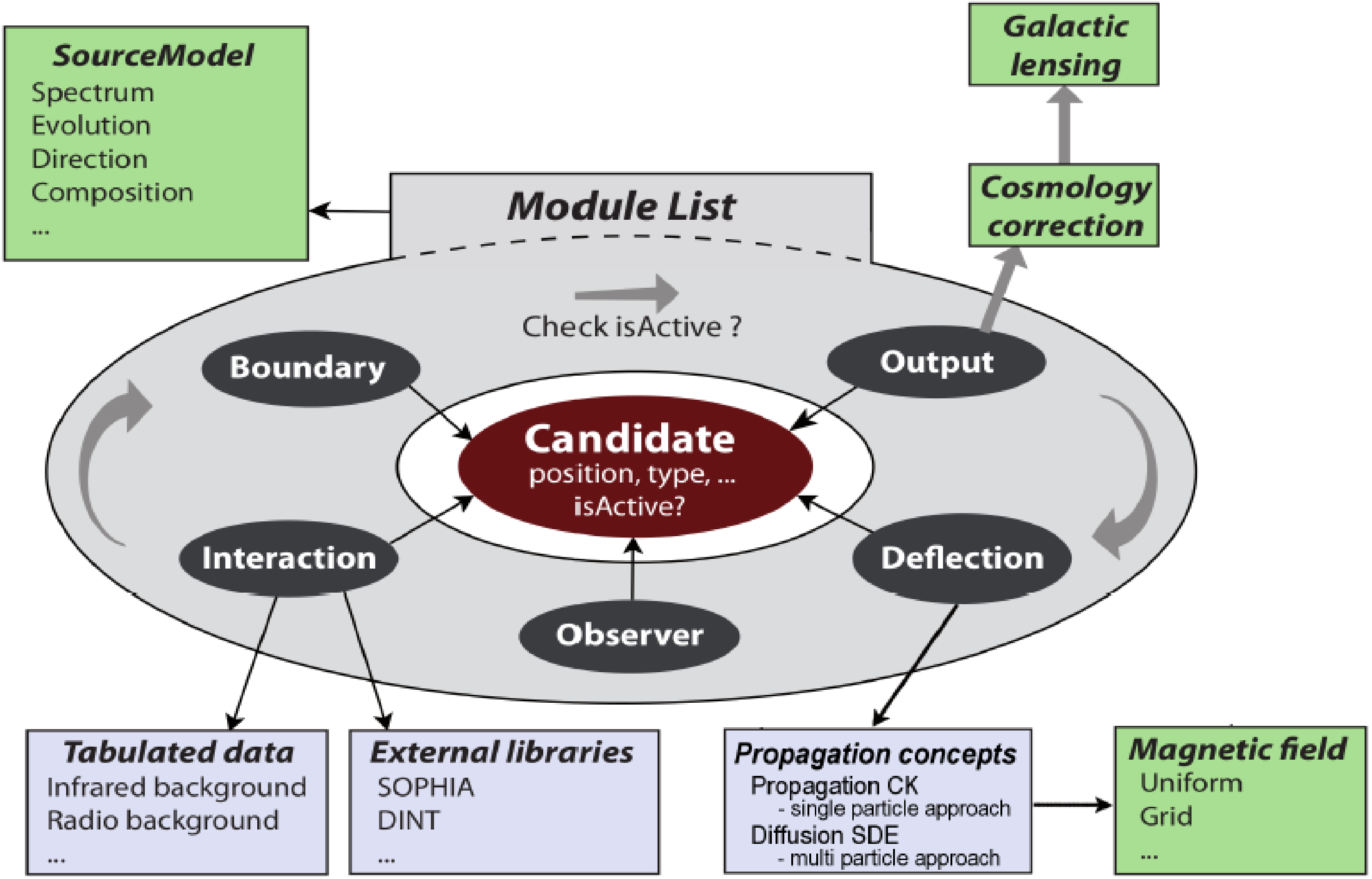
Simulation

CRPropa 3.2 – a modular structure



Transport

- Propagation of Cosmic Rays using the Parker transport equation
- Taking advantage of the collective behavior of the CRs

$$\frac{\partial n}{\partial t} + \vec{u} \cdot \nabla n = \nabla \cdot (\hat{\kappa} \nabla n) + \frac{1}{p^2} \frac{\partial}{\partial p} \left(p^2 \kappa_{pp} \frac{\partial n}{\partial p} \right) + \frac{p}{3} \nabla \cdot \vec{u} \frac{\partial n}{\partial p} + S$$

- Anisotropic diffusion in homogeneous background, including advection and adiabatic losses

Transport

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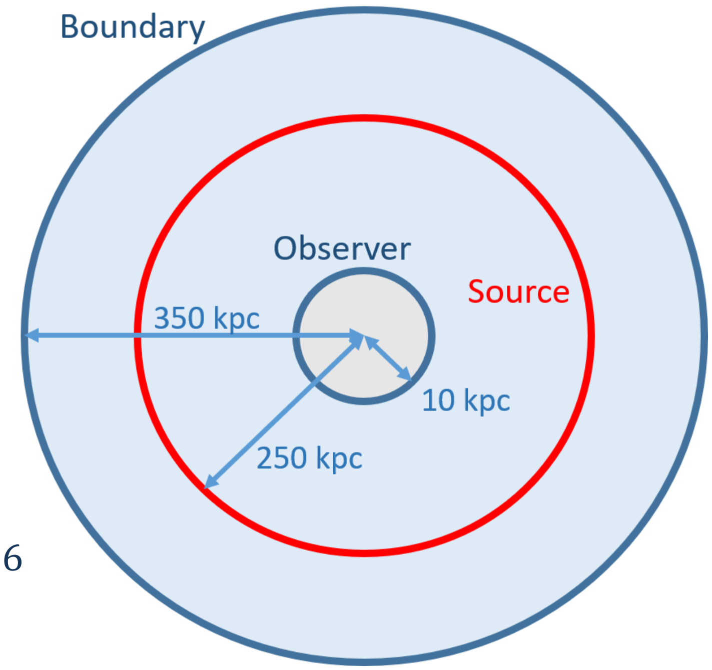
- Anisotropic diffusion in homogeneous background, including advection and adiabatic losses

Source

- Symmetry
  - Radial / Archimedean spiral
- Diffusion
  - D$\propto E^{\delta}$; $D_0 = 10^{28} \frac{cm^2}{s}$
- Advection
  - $v_0 = 600 \frac{km}{s}$; $r_0 = 250\ kpc$
- Spectrum: $\frac{\mathrm{dN}}{\mathrm{dE}} \propto E^{-2.0}; E = 10^{15} - 10^{16}$



The wind model – radial component

- Rise of the wind not included → constant velocity
- Analytically smooth shock front
- Wind drops with $1/r^2$

$$v_r(r) = v_0 \left[1 + \frac{\left(\left(\frac{r}{r_0} \right)^2 - 1 \right)}{1 + e^{\frac{r_0 - r}{\lambda}}} \right]$$

The wind model – radial component

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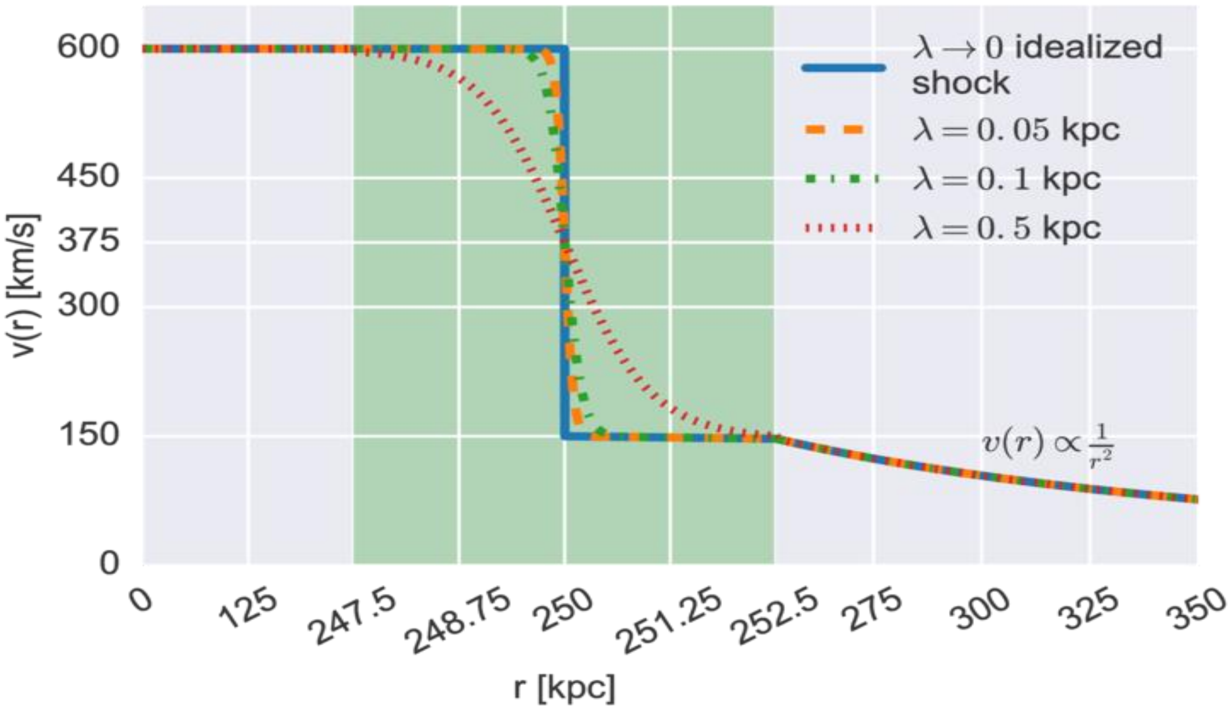

RESULTS

Adiabatic energy change

$$\frac{\partial n}{\partial t} + \vec{u} \cdot \nabla n = \frac{p}{3} \nabla \cdot \vec{u} \frac{\partial n}{\partial p} + S$$

Cooling ($\nabla \cdot \vec{u} > 0$)

- Expansion of the plasma → energy loss

Heating ($\nabla \cdot \vec{u} < 0$)

- Compression of the plasma → energy gain

 Lukas.Merten@rub.de

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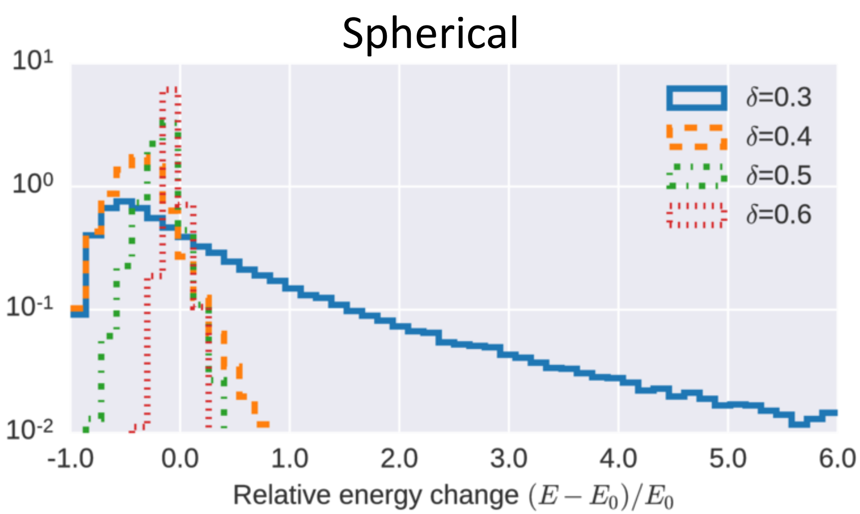
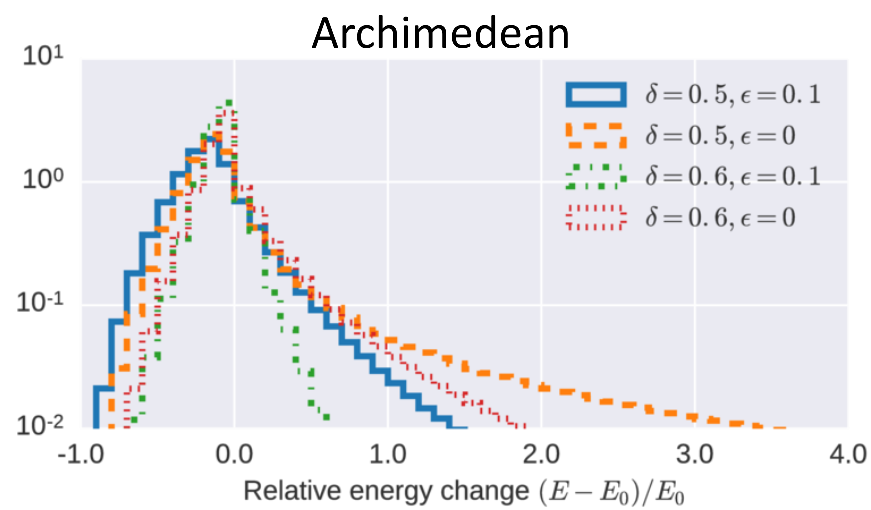
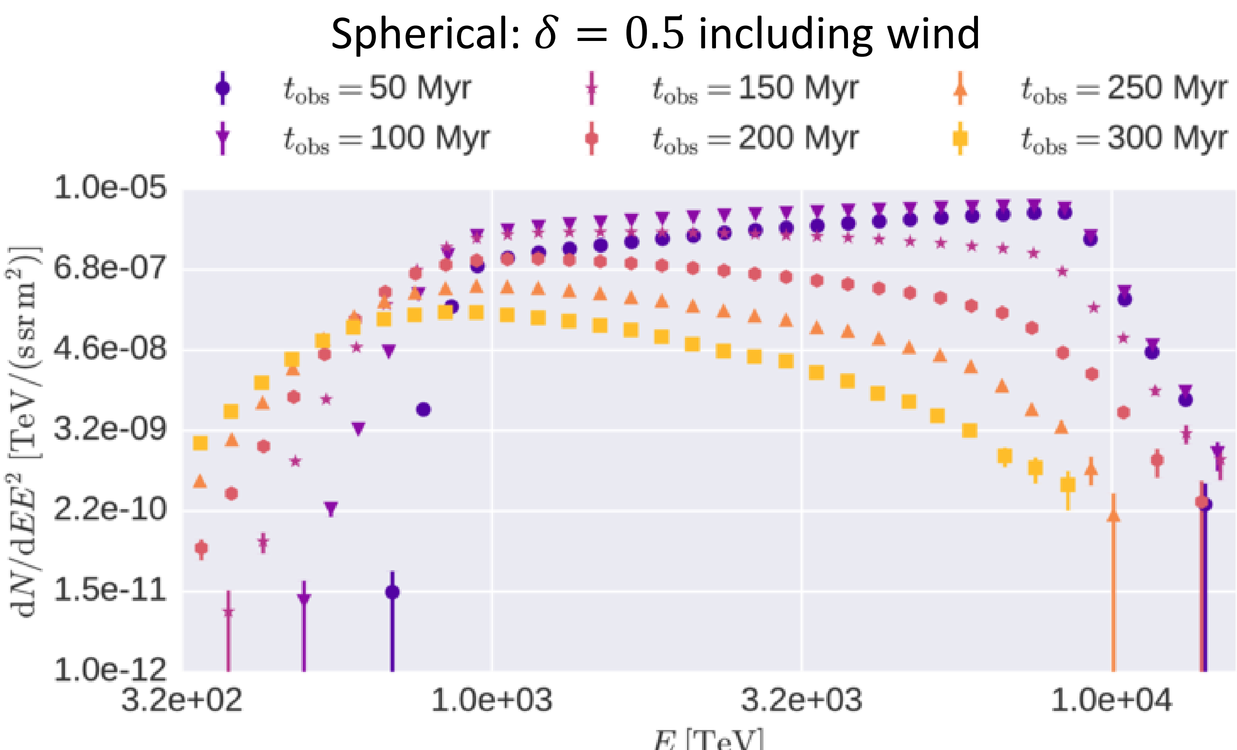




Energy Spectrum

- A significant change in the spectral shape is visible
- Three regions can be identified, dominated by:



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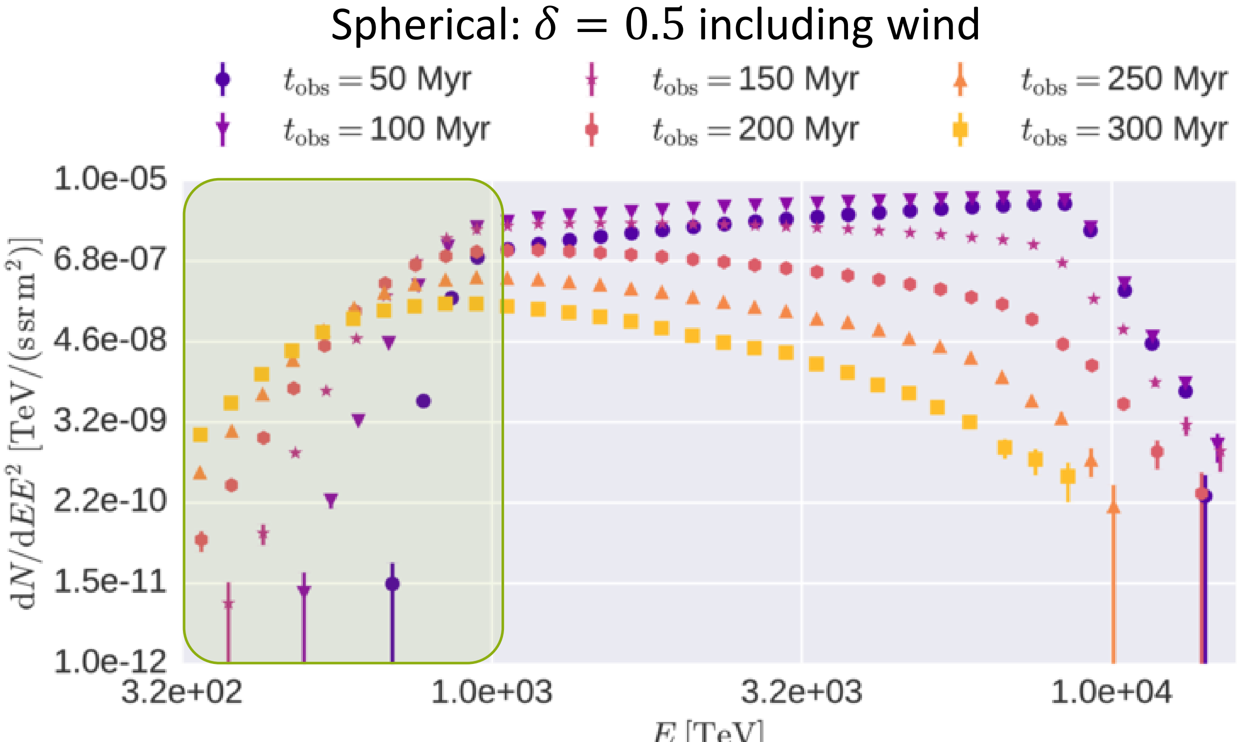
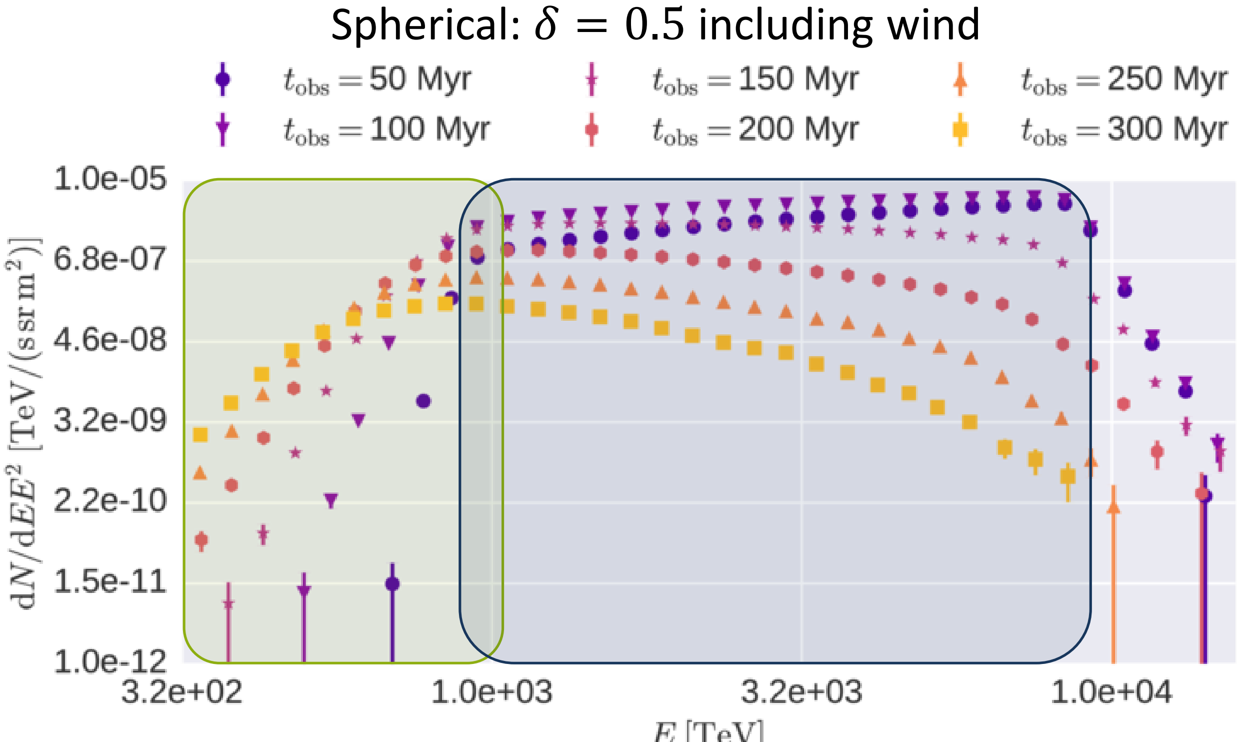
Energy Spectrum

- A significant change in the spectral shape is visible
- Three regions can be identified, dominated by:
  - 1) cooling



Energy Spectrum

- A significant change in the spectral shape is visible
- Three regions can be identified, dominated by:
  - 1) cooling, 2) diffusion



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Energy Spectrum

- A significant change in the spectral shape is visible
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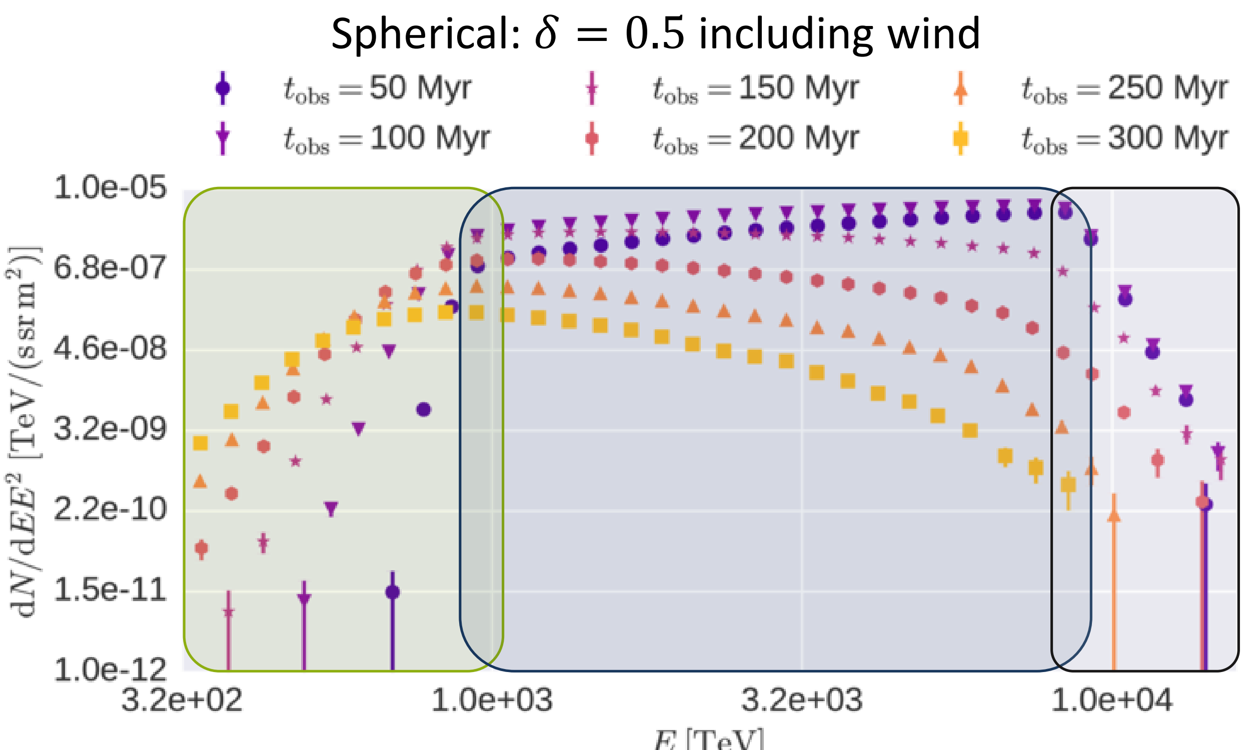

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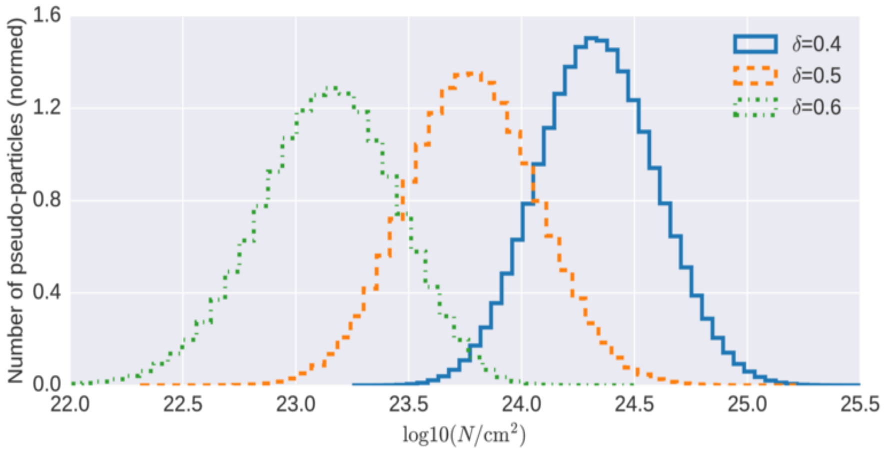
Neutrino Flux

- Hadronic interaction: $p_{\mathrm{CR}} + p_{\mathrm{target}} \rightarrow \pi^+ \rightarrow e^+ + \nu_\mu + \widetilde{\nu_\mu} + \nu_e$
- $n_{\mathrm{target}} \propto \frac{1}{r^2}$ → Use accumulated column density to calculate flux
  - Assumption: All neutrinos are produced at $r_{\mathrm{obs}} = 10$ kpc

Column density



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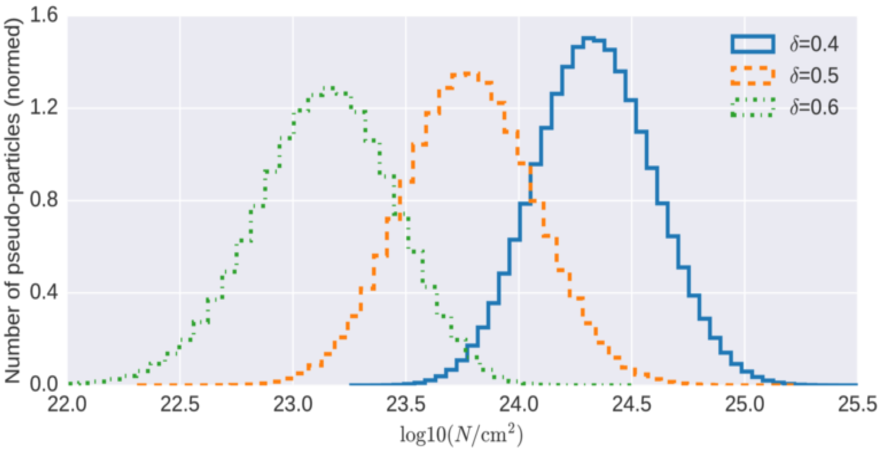
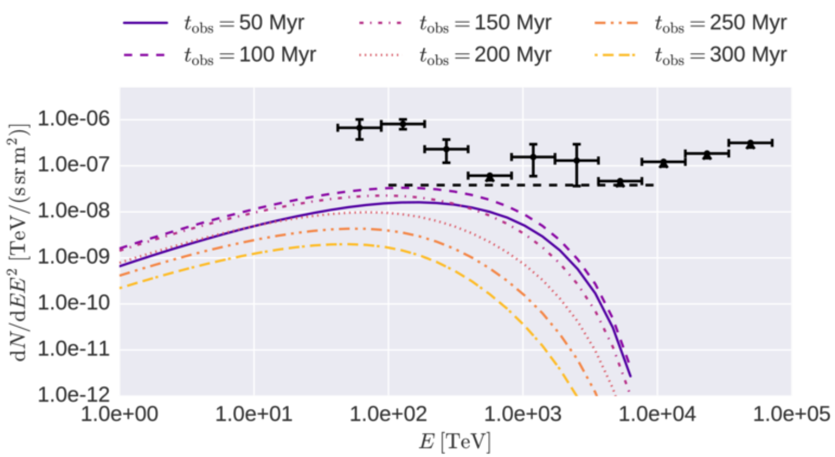
Column density



Flux for $\delta = 0.5$



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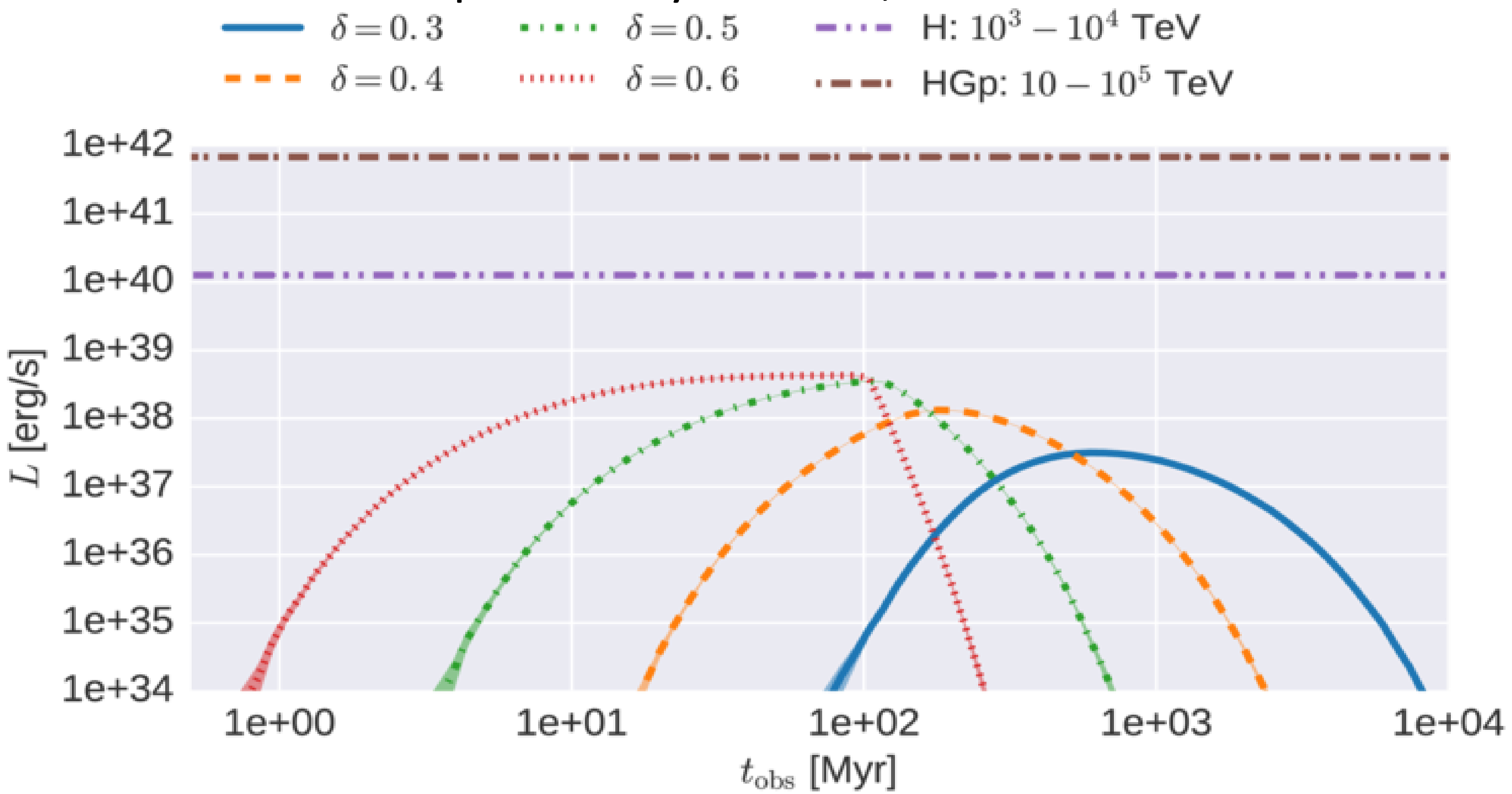
Total proton flux

Spherical symmetric, no wind



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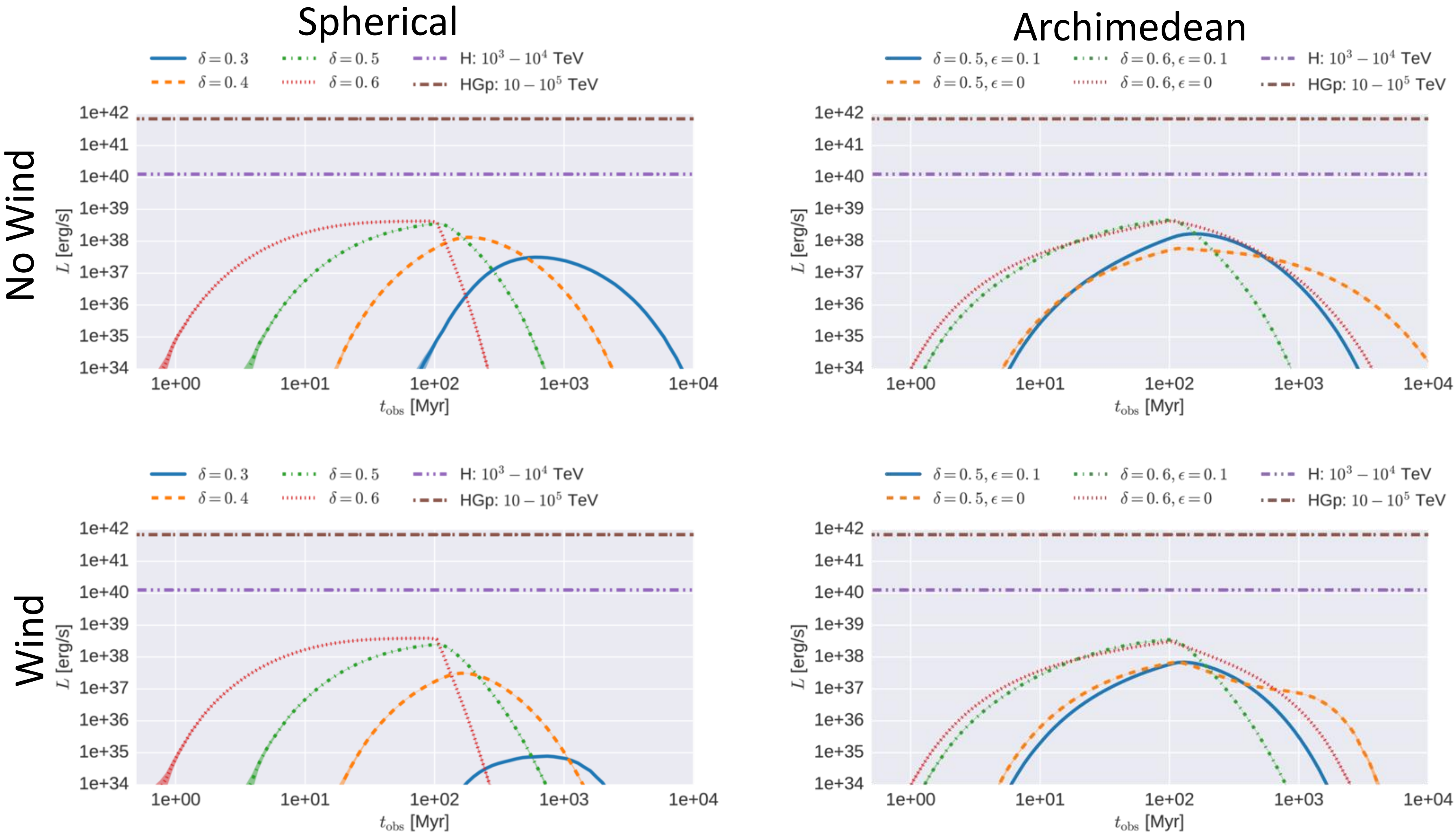
Total proton flux



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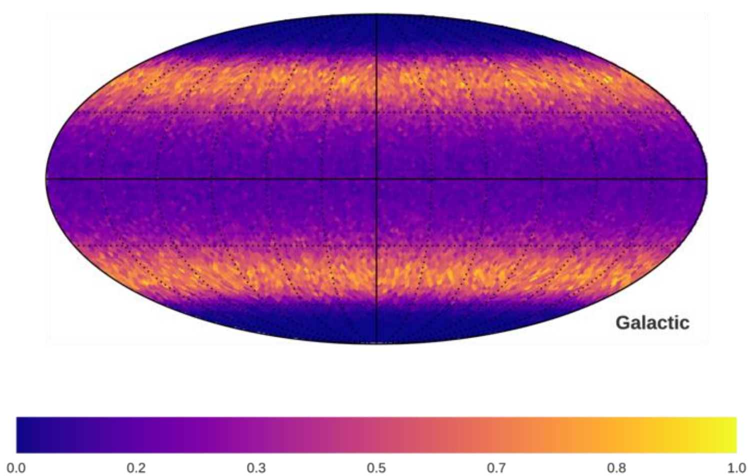
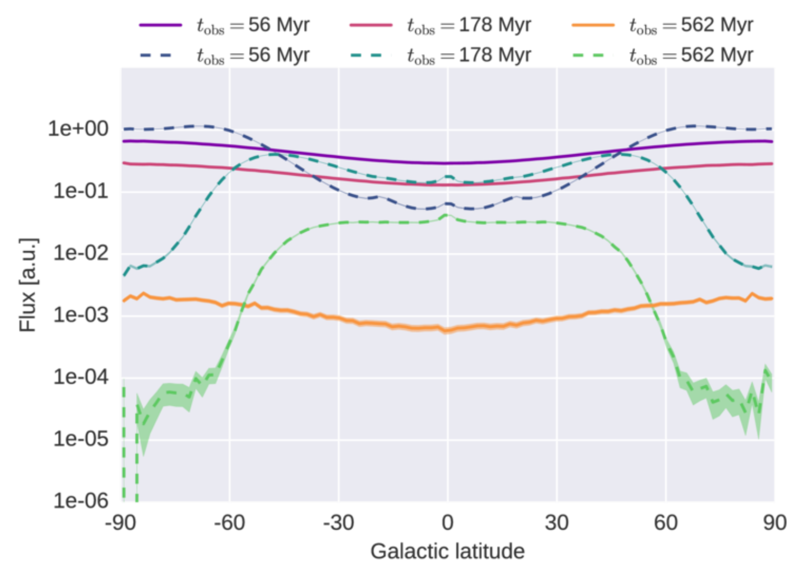
Arrival Direction – Archimedean spiral

- Different field line length → double ring structure
- Maxima shift from the poles to the equator
- Perpendicular diffusion ($\kappa_\perp = 0.1\kappa_\parallel$) washes the structure out

Skymap in Galactic coordinates



Galactic latitude (Θ distribution)



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SUMMARY / OUTLOOK

Summary

- The expected spectra depend strongly on the model → compatible with observations
- (Stable) Neutrino flux is below IceCube diffuse measurements
- Anisotropy in the is to high
- Proton total luminosity is too low

→ CRs from the GWTS cannot be the sole source in the shin region

Outlook

- Further propagation of the CRs to Earth
- Restrict the wind to a fraction of the sphere
- Simulate the neutrino production
- Look at CRs leaving the wind termination shock
  → Starburst Galaxies

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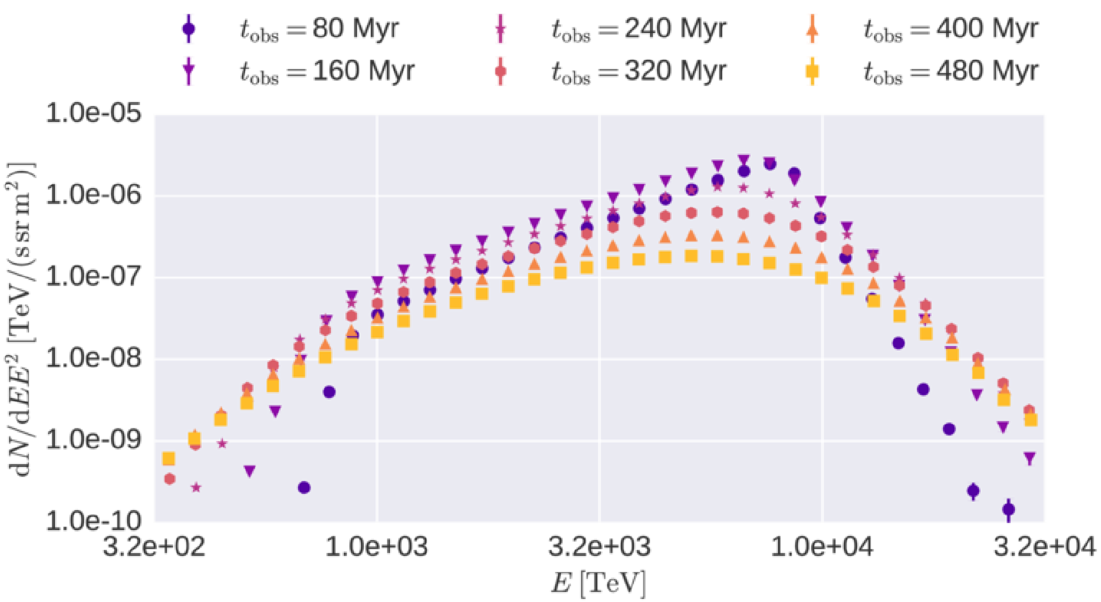
BACKUP

Energy Spectrum

- A significant change in the spectral shape is visible
- Three regions can be identified, dominated by:
  - 1) cooling, 2) diffusion, and 3) heating

Archimedean spiral: $\delta = 0.5, \kappa = 0.1$, including wind



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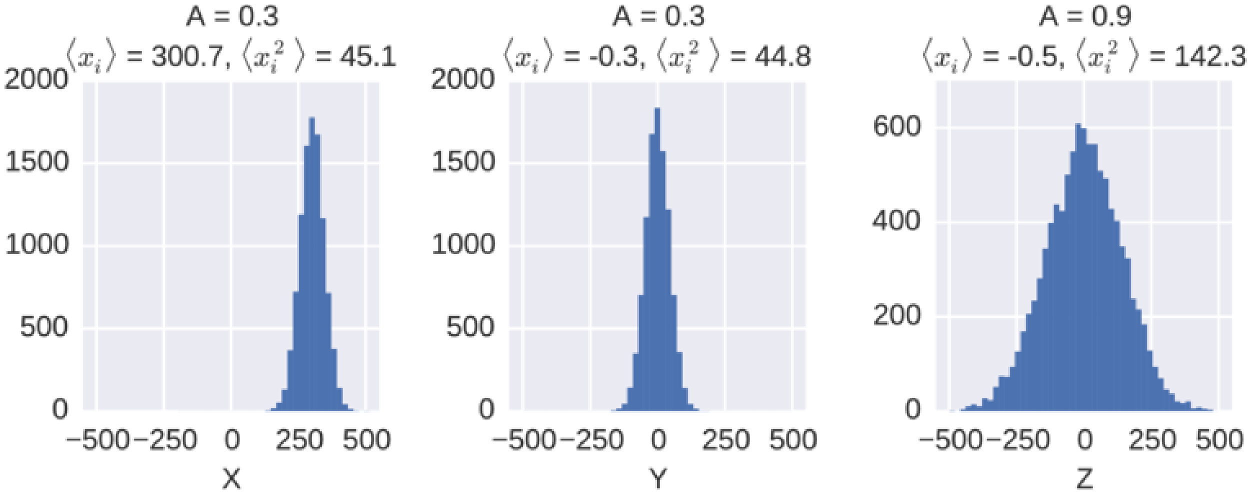
Transport

- Propagation of Cosmic Rays using the Parker transport equation
- Taking advantage of the collective behavior of the CRs

$$\frac{\partial n}{\partial t} + \vec{u} \cdot \nabla n = \nabla \cdot (\hat{\kappa} \nabla n) + \frac{1}{p^2} \frac{\partial}{\partial p} \left(p^2 \kappa_{pp} \frac{\partial n}{\partial p} \right) + \frac{p}{3} \nabla \cdot \vec{u} \frac{\partial n}{\partial p} + S$$

Anisotropic diffusion in homogeneous background including advection



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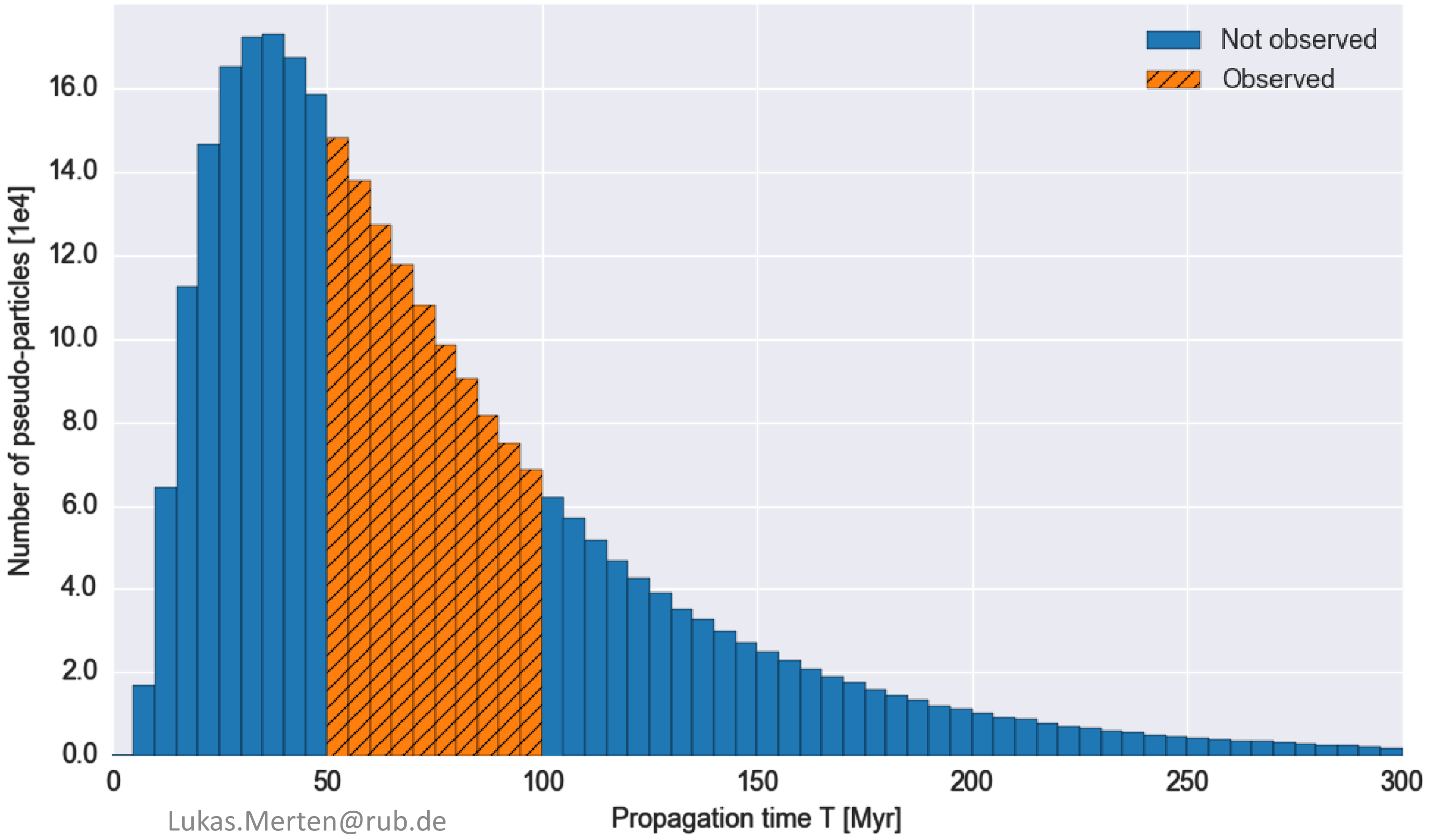
Green‘s method

$S_{\mathrm{burst}} = S_0 \delta(r_0) \delta(t - t_0)$

$S_{\mathrm{finite}} = \tilde{S}_0 \Theta(t - t_0) \Theta(\mathrm{t}_1 - \mathrm{t}_0)$; Example: $t_{\mathrm{obs}} = 100$ Myr; $\Delta t = t_1 - t_0 = 50$ Myr



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Model Assumption

Symmetry

- Radial / Archmedean spiral

Diffusion

- D∝ E^{δ}
- $D_0 = 10^{28}\,\frac{cm^2}{s}$

Advection

- $v_0 = 600\,\frac{km}{s}$
- $r_0 = 250\ kpc$

Boundaries

- $r_{\mathrm{obs}} = 10\ kpc$
- $r_{\mathrm{loss}} = 350\ kpc$

Simulation details

- $N = 10^7 - 8.5 \cdot 10^8$
- $\tau_{\mathrm{CPU}} = O(10)h - O(1000)h$

OBSERVATION

Observation – Energy Spectrum

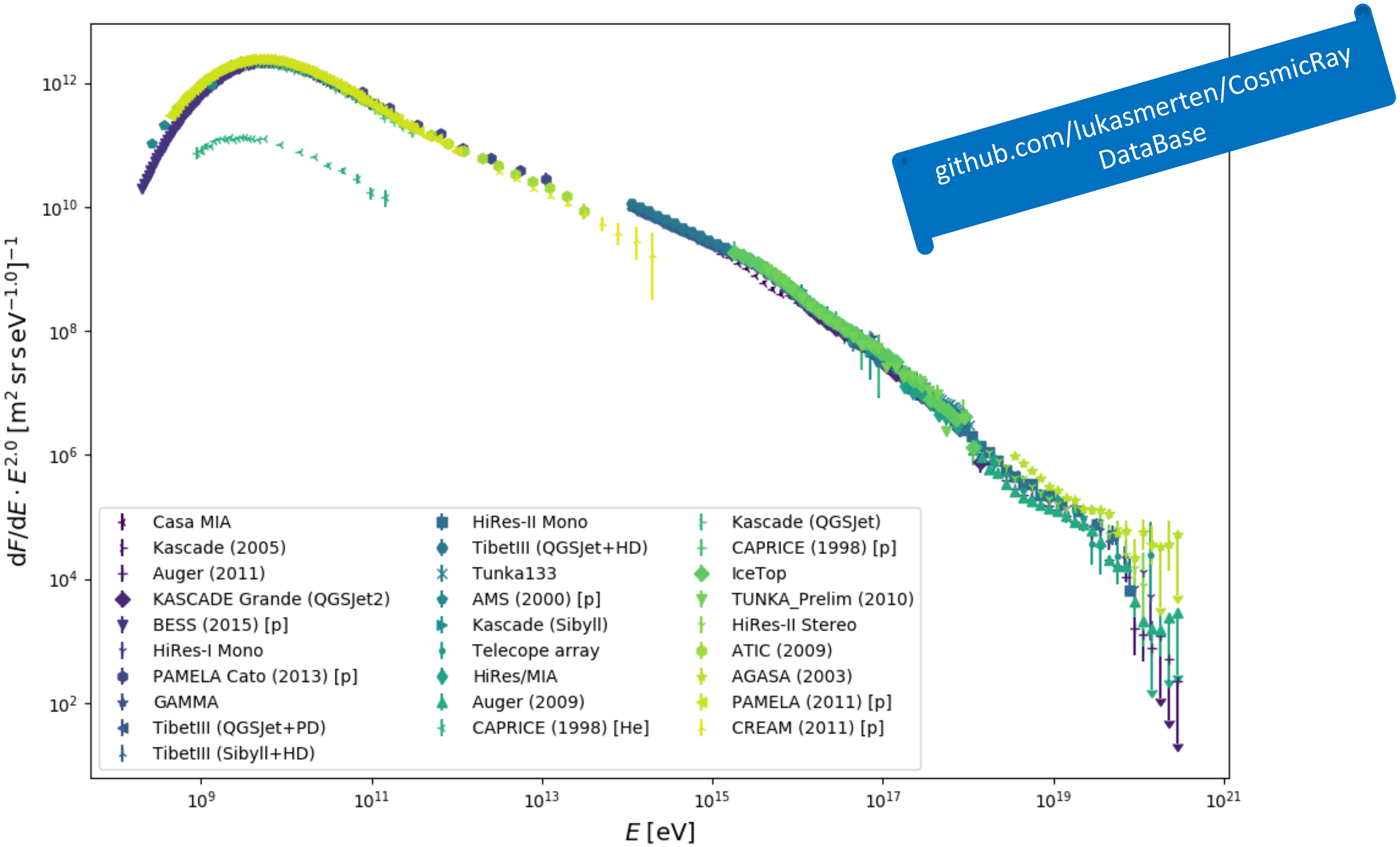
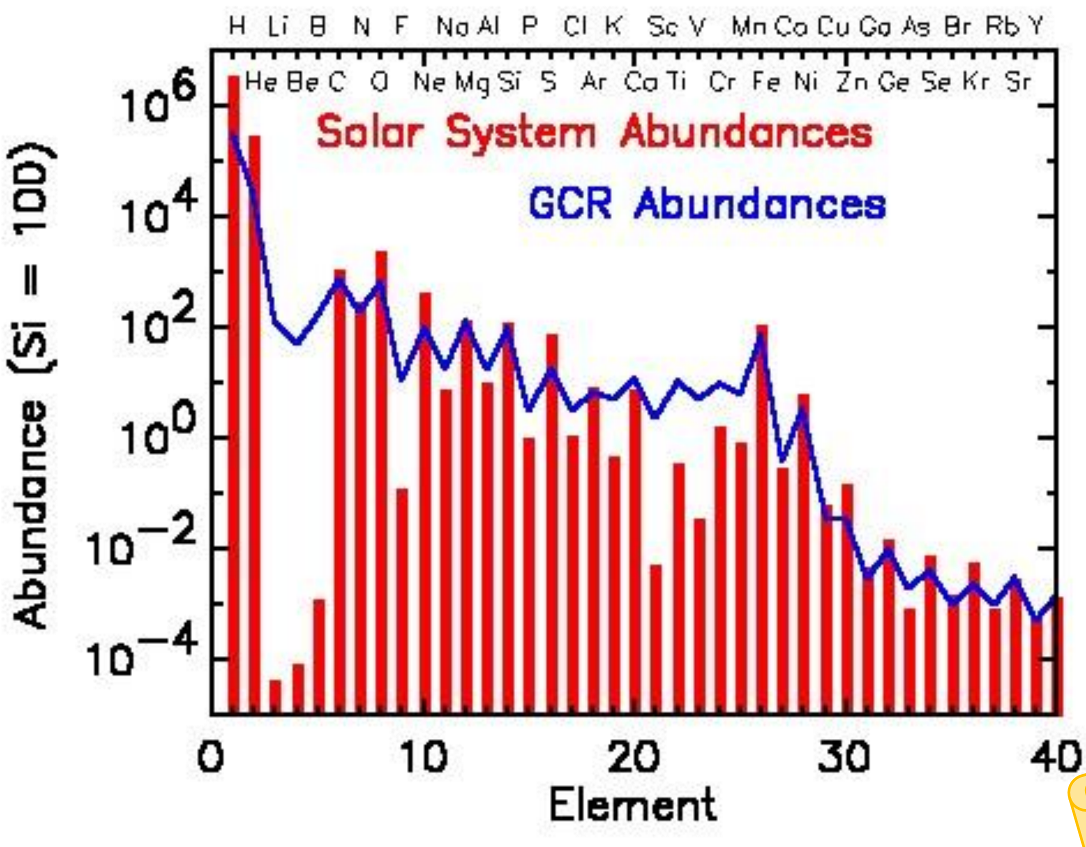


Fig 2. The energy spectrum of Cosmic rays.

Observation - Composition



NASA. Imagine the Universe. access: (23.02.2015)

Fig 3. Boron to carbon ratio as a measure for the column depth.

Observation – B/C ratio

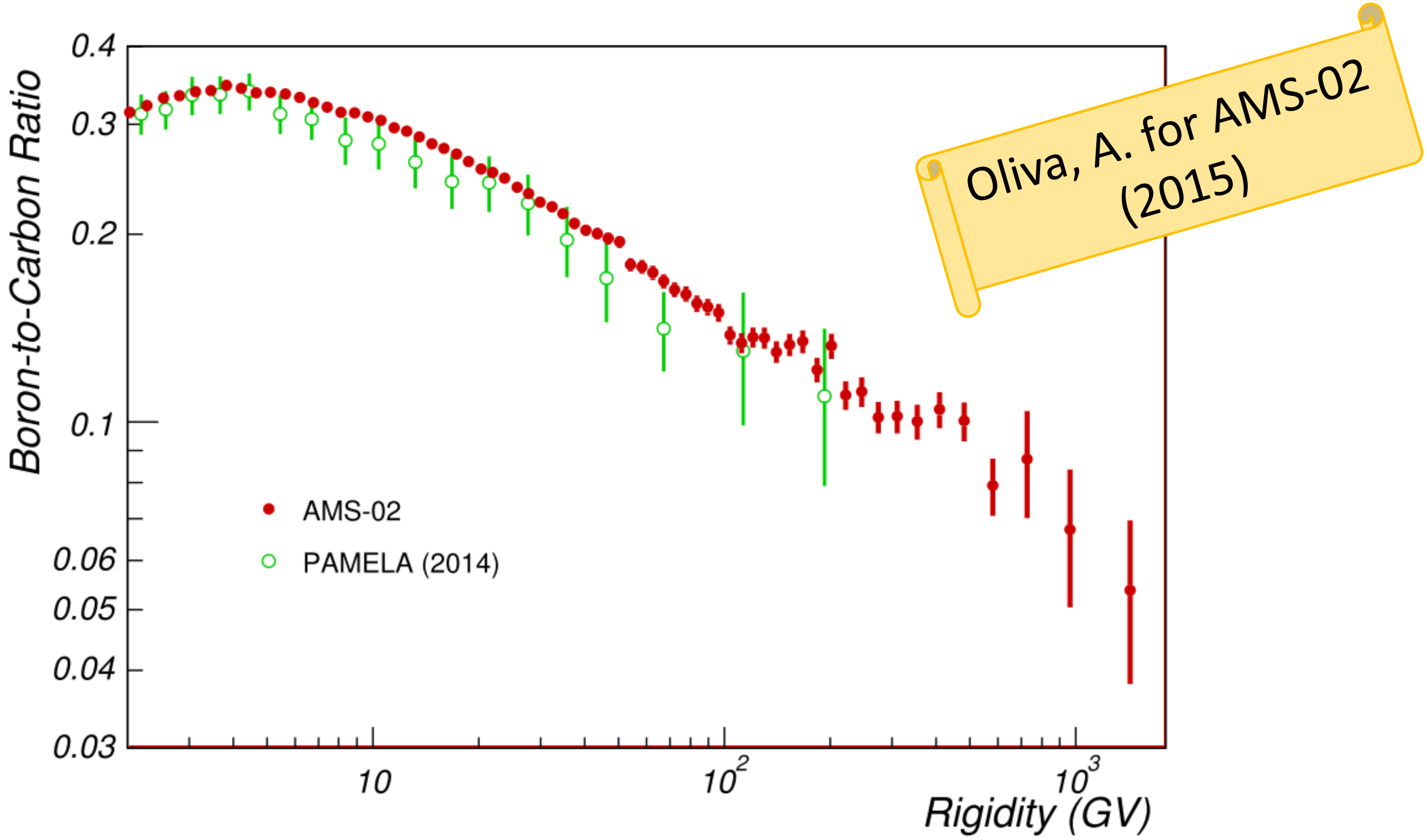


Fig 3. Boron to carbon ratio as a measure for the column depth.

Observation – Arrival Directions

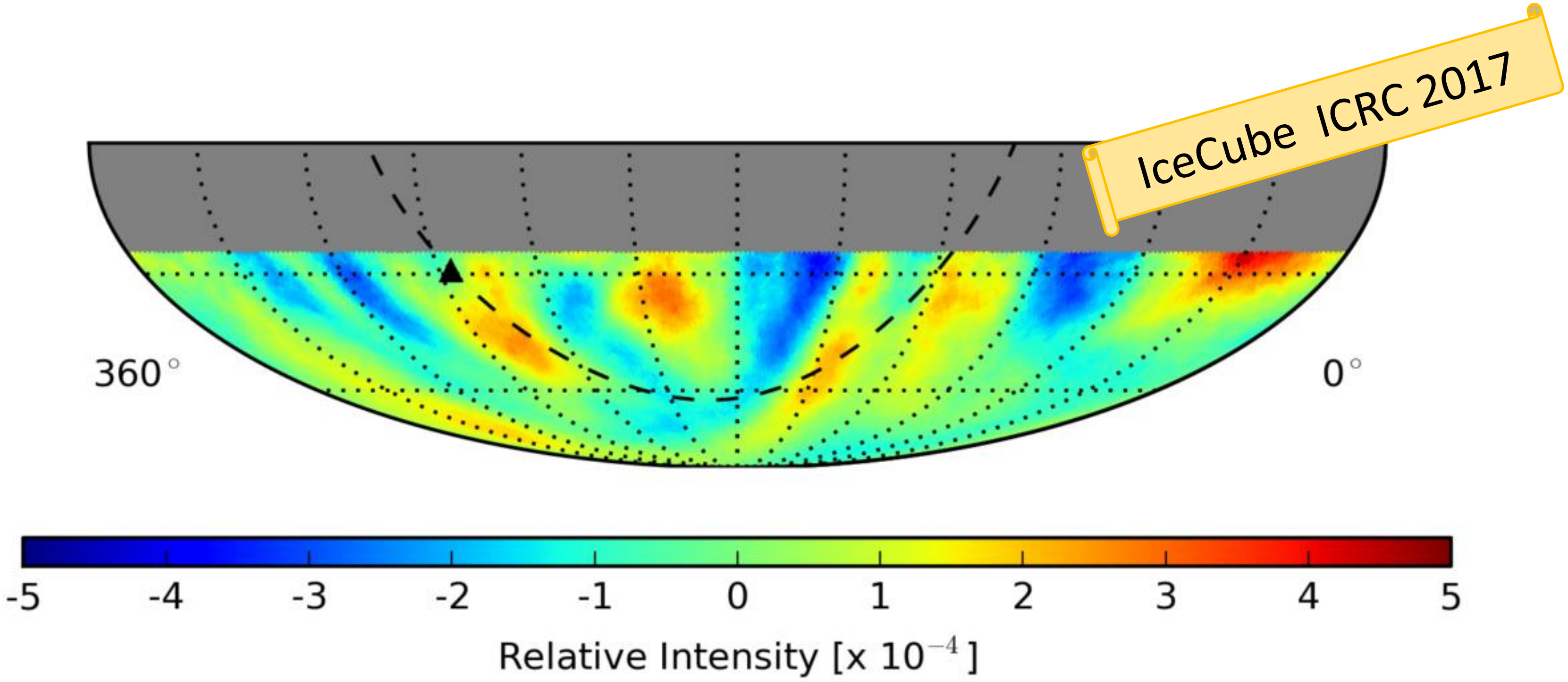


Fig 4. Cosmic ray arrival anisotropy at a median energy E=20 TeV after the subtraction of the best fit dipole and quadrupole.

SDE - MATH

Merten et al. (2017), arXiv:1704.07484

2 Propagation Models

Singleparticle picture

- Solving the equation of motion
- Backtracking and Reweighting of results is possible

Numerics

Multiparticle picture

- Solving the transport equation
- Collective behavior, e.g. advection and diffusion is easy to implement

Physics

Stochastic Differential Equations

Lukas.Merten@rub.de

Stochastic Differential Equation

Langevin Equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} = a(x,t) + b(x,t)\xi(t)$$

Stochastic Integral Equation

$$x(t) = x(0) + \int_{t_0}^{t} a[x(s), s]\ \mathrm{d}s + \int_{t_0}^{t} b[x(s), s]\ \mathrm{d}W(s)$$

- These equations can be treated mathematically consistently.
- Numerical algorithm to solve them are available.

From Fokker-Planck Equ. to SDE

General Fokker-Planck Equation

$$\frac{\partial n(x,t;y,t')}{\partial t} = -\sum_i \frac{\partial}{\partial x_i}[A_i(x,t)n(x,t;y,t')] + \frac{1}{2}\sum_{i,j}\frac{\partial^2}{\partial x_i \partial x_j}[B_{ij}(x,t)n(x,t;y,t')]$$

Corresponding Stochastic Differential Equation

$$\mathrm{d}r_\nu = A_\nu\,\mathrm{d}t + D_{\nu\mu}\,\mathrm{d}\omega^\mu$$

Calculation of stochastic tensor D

$$(\kappa + \kappa^t) = DD^\dagger$$

In the case of decoupled momentum und spatial operators
And diagonal diffusion tensor

$$D_{ij} = \delta_{ij}\sqrt{2\kappa_{ij}}, \qquad D_{qq} = \sqrt{2\kappa_{qq}}$$

Integration scheme

$$\vec{x}_{n+1} = \vec{x}_n + D_r\,\Delta\vec{\omega}_r$$
$$= \vec{x}_n + \left(\sqrt{2\kappa_{||}}\,\eta_{||}\,\vec{e}_t + \sqrt{2\kappa_{\perp,1}}\,\eta_{\perp,1}\,\vec{e}_n + \sqrt{2\kappa_{\perp,2}}\,\eta_{\perp,2}\,\vec{e}_b\right)\sqrt{h}$$

- Magnetic field line implementation (e.g. JF12 field) is not trivial.
- Calculation of the local trihedron is the crucial part.

- Use adaptive field line integration → e.g. Cash-Karp algorithm.

$$\vec{r}_{end} = \vec{r}_{start} + \int_0^L \vec{B}/B\,\mathrm{d}s \qquad \vec{r}_{end} = \vec{r}_0 + \sum_{j=0}^{2^n-1} \int_{2^{-n}L\,j}^{2^{-n}L\,(j+1)} \vec{v}(s)\,\mathrm{d}s$$

SDE and FPE

Ito's lemma leads to equivalence between stochastic differential equation (SDE) and Fokker-Planck transport equation (FPE)

$$\frac{\partial \Psi(r,t)}{\partial t} = \nabla \cdot \left(D \cdot \nabla \Psi(r,t)\right) + S(r,t)$$

$$dr = A\,dt + B\,d\omega \qquad with: d\omega = \eta\sqrt{dt}$$

$$B = \sqrt{2 \cdot D,}\ A = 0$$

Diffusion tensor is diagonal in frame with $\vec{B} = B_0 \cdot \overrightarrow{e_z}$

$$D = \begin{bmatrix} \kappa_{perp,1} & 0 & 0 \\ 0 & \kappa_{perp,2} & 0 \\ 0 & 0 & \kappa_{par} \end{bmatrix} \qquad with: \kappa_{perp,1} = \kappa_{perp,2}:$$

CRPropa

CRPropa 3.2 – a rich toolbox

Deflection
- SimplePropagation
- PropagationCK
- DiffusionSDE

Nucleon-Interacion
- ElectronPairProduction
- PhotoPionProduction
- PhotoDisintegration
- NuclearDecay

EM-Interactions
- EM(Double/Triple)-PairProduction
- EMInverseCompton-Scattering

General Interactions
- Redshift
- SynchrotronRadiation
- AdiabaticCooling

Boundaries/ Thresholds
- MaximumTrajectory-Length
- MinimumEnergy
- CubicBoundary
- SphericalBoundary
- …

Observer
- ObserverSmallSphere
- ObserverTracking
- ObserverPoint
- ObserverDetectAll
- ObserverTimeEvolution
- …

Output
- ShellOutput
- TextOutput
- HDF5Output
- ParticleCollector

Others
- PerformanceModule

PERFORMANCE

Performance

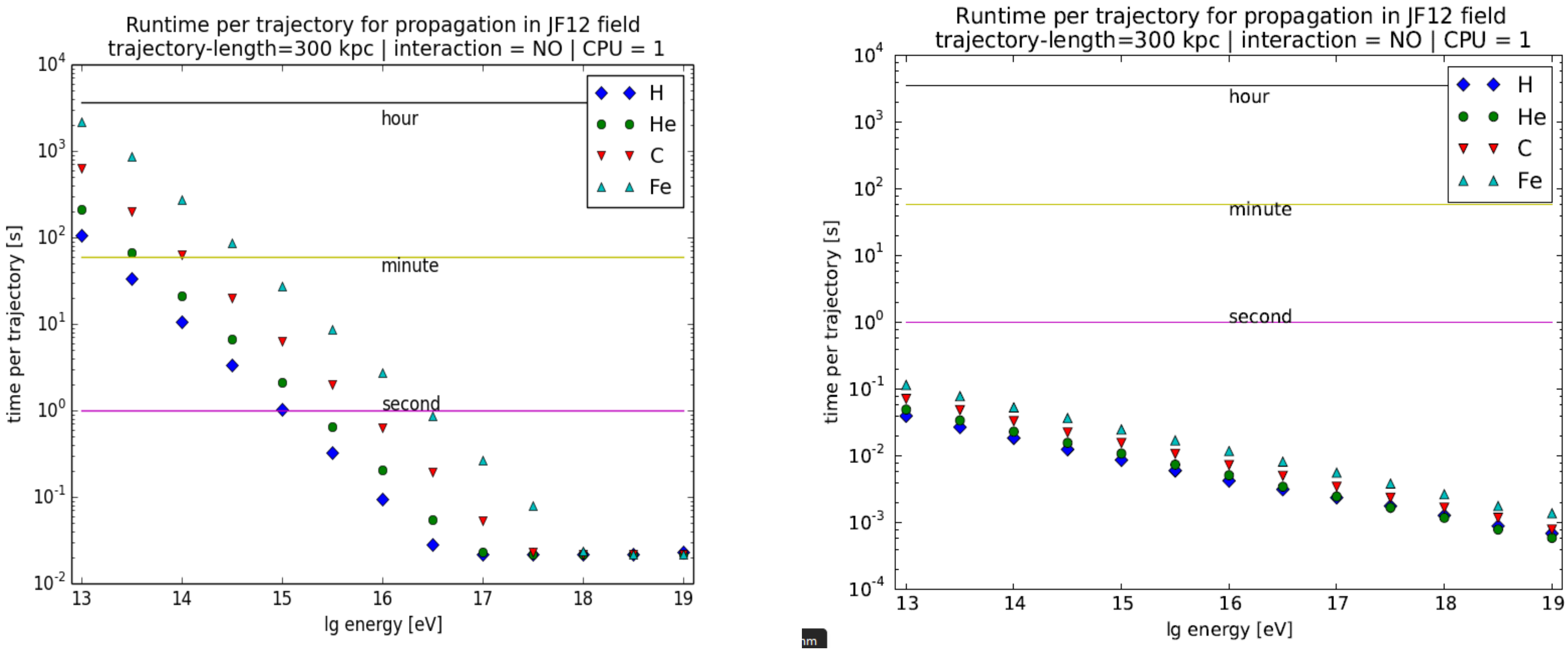


Fig 15: Comparison of computation times. Conventional CRPropa3 (left) and propagation with the DiffusionModule (right)

PROBLEMS WITH THE JF12-FIELD

Problem: Fixed step length

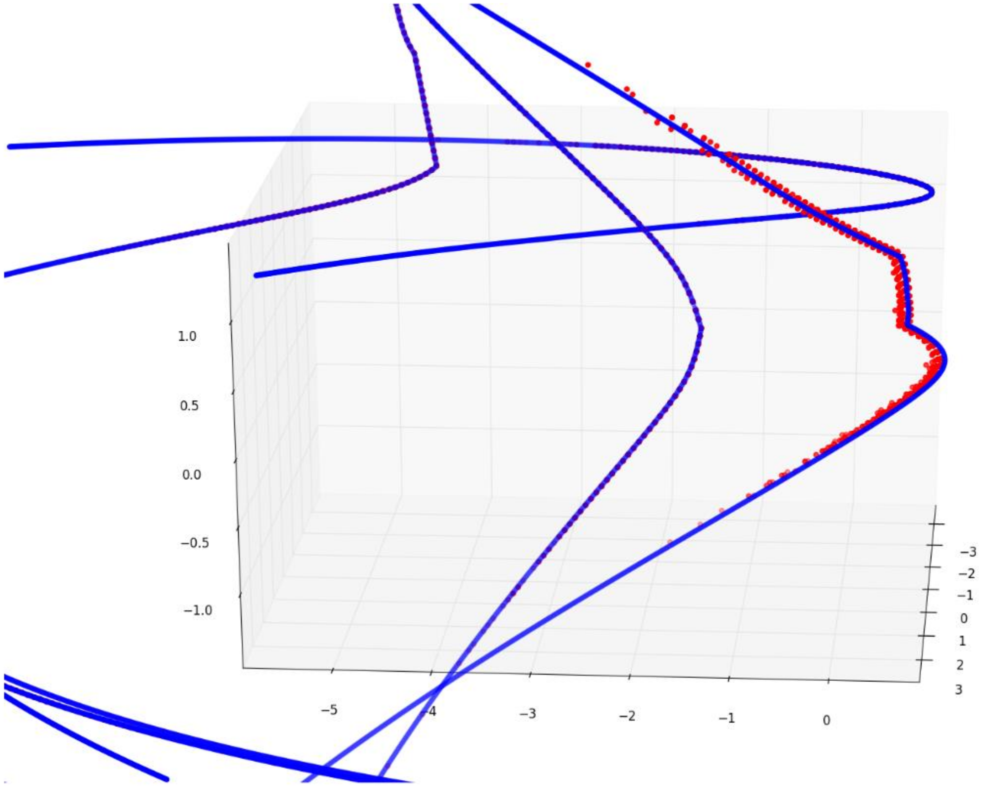


Fig 19: Magnetic field line (blue) and end position after diffusion process (red dots).

Deviation vectors

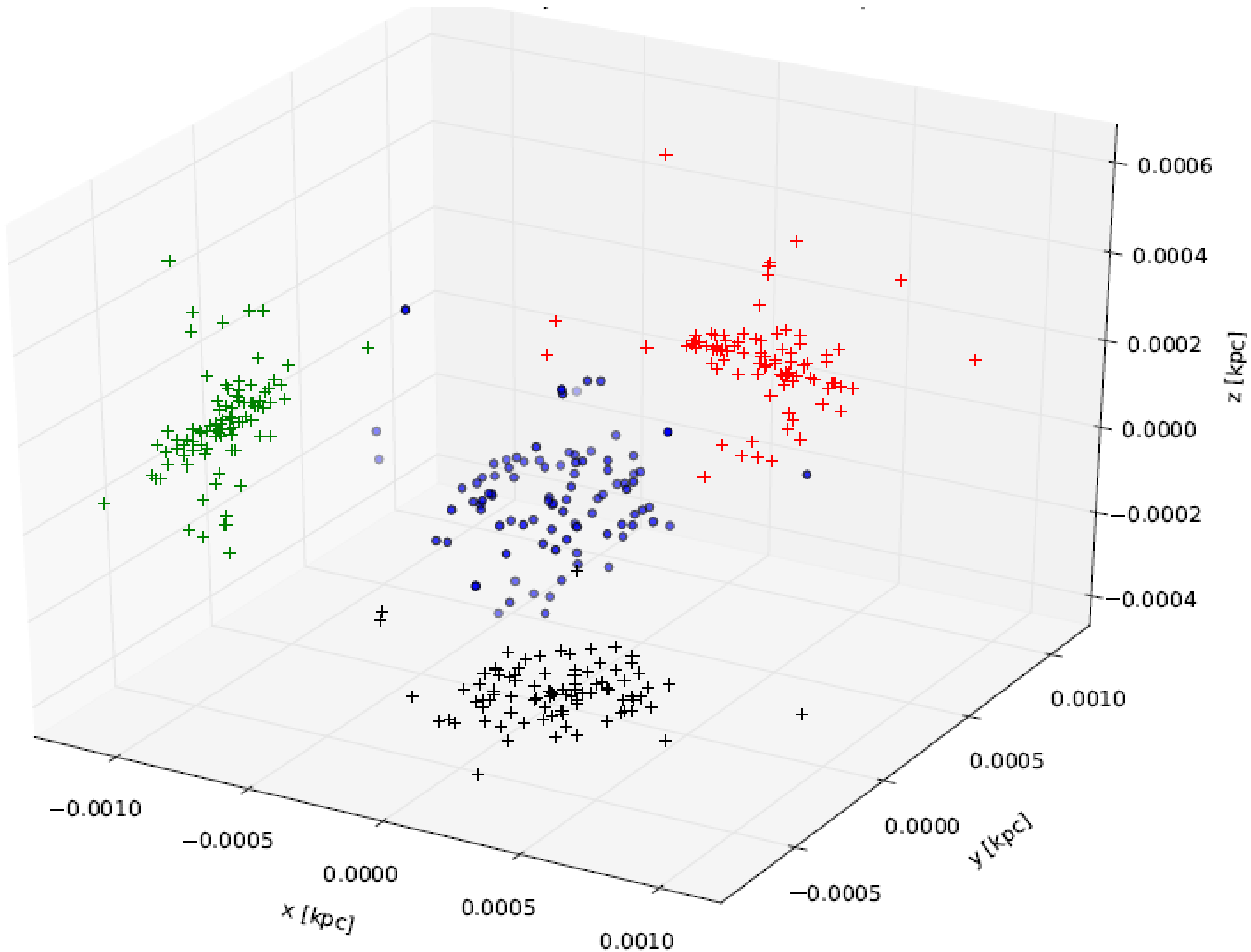


Fig 20: Ninty smallest deviation vectors for E=10TeV

Deviation from field line II

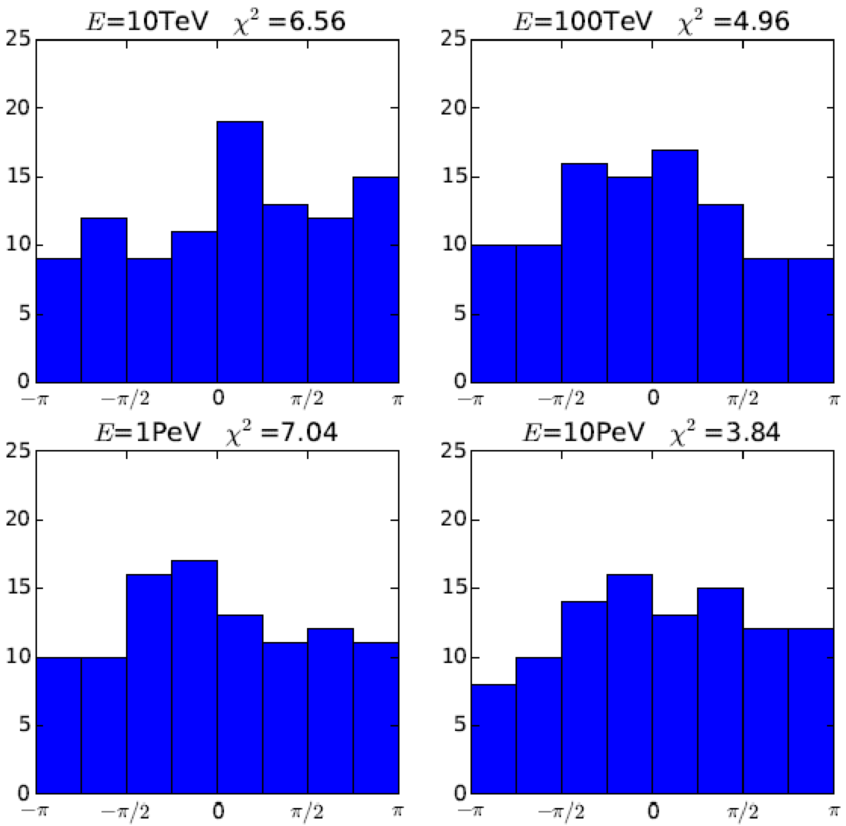


Fig 21: Deviation from ideal trajectory is uniformly distributed in Φ.

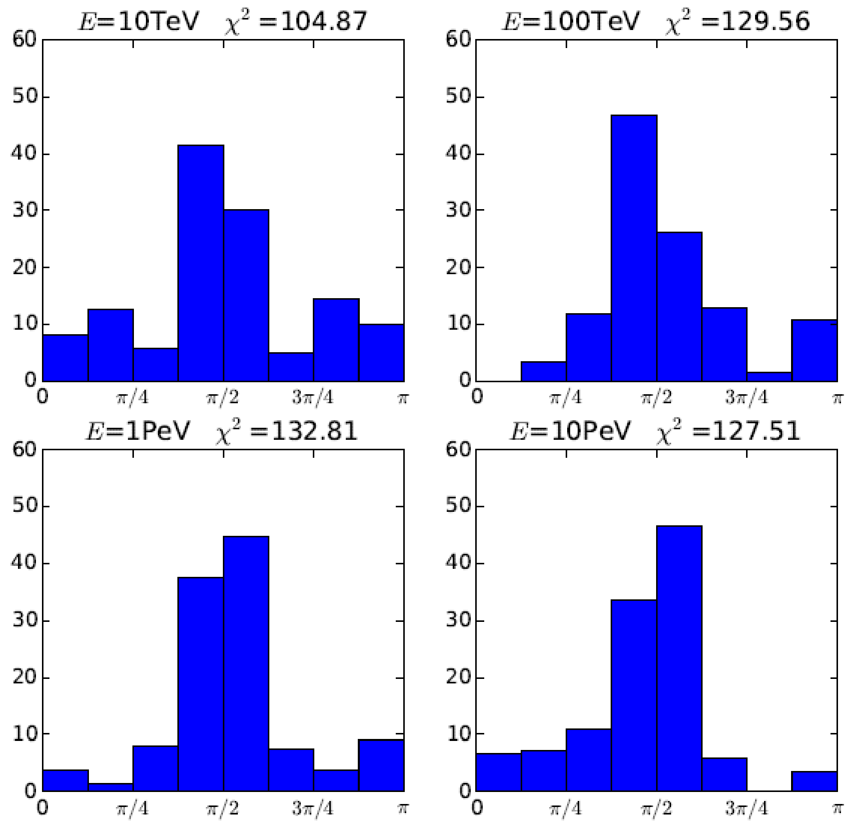


Fig 22: Deviation from ideal trajectory peaks around the plane perpendicular to magnetic field line.

VALIDATION

Validation I

First test of the diffusion in a homogeneous magnetic background field.
A simple anisotropic diffusion tensor is implemented.

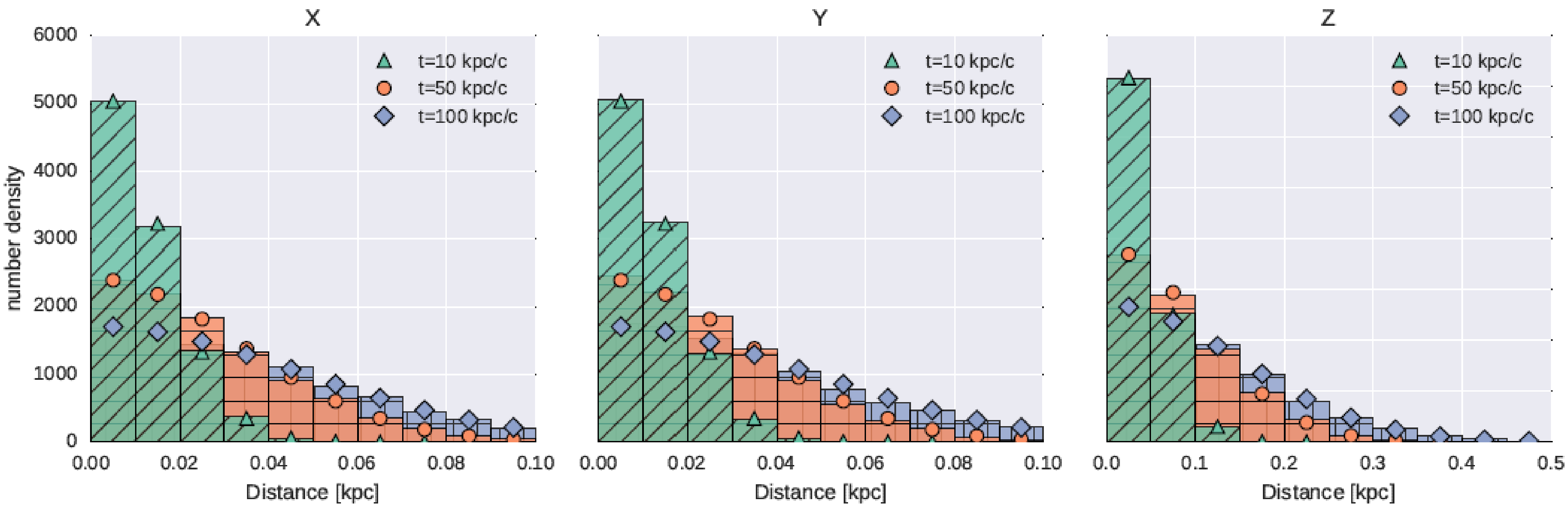


Fig 7. The algorithm reproduces the expected analytic results (simulation-barplot, theory-solid lines).

Stationary Test I

Stationary equation of anisotropic diffusion

$$-\nabla \cdot (\hat{\kappa} \nabla n(\vec{r})) = s(\vec{r})$$

Not possible?

Source term

$$s(\vec{r}) = \frac{4}{\pi^2} \left(\frac{\kappa_{xx}}{4R^2} + \frac{\kappa_{zz}}{2H^2} \right) \cdot \cos\left(\frac{x\pi}{2R}\right) \cos\left(\frac{y\pi}{2R}\right) \cos\left(\frac{z\pi}{2H}\right)$$

Indirect solution

$$\frac{\partial n}{\partial t} = \nabla \cdot (\hat{\kappa} \nabla n) + s(\vec{r}) \delta(t - t_i)$$

$$n_{sim}(\vec{r}) = \sum_i n(t_i) h_i w$$

Stationary Test II

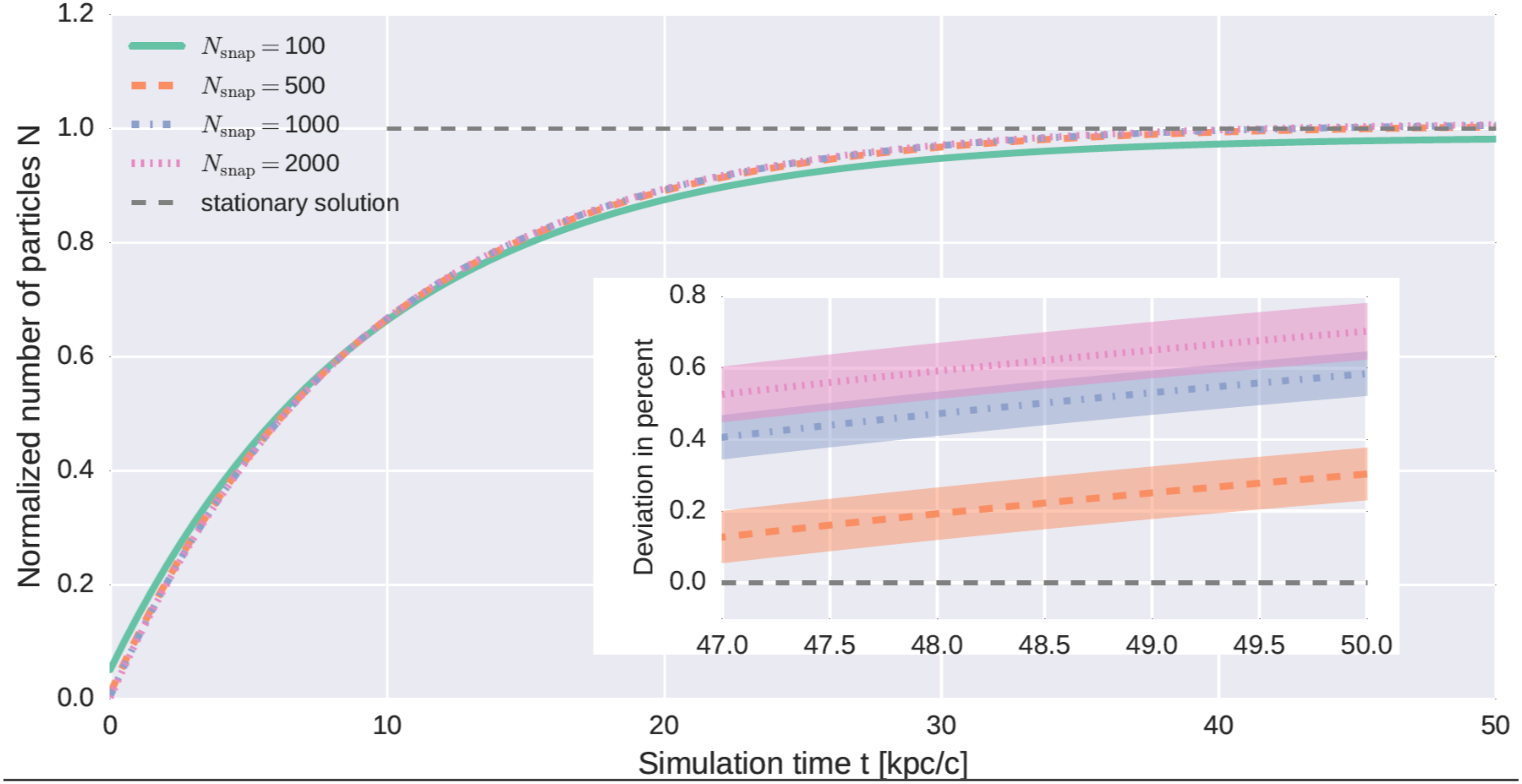


Fig 8. Total number density depending on maximum integration time for different numbers of snapshots.

Stationary Test III

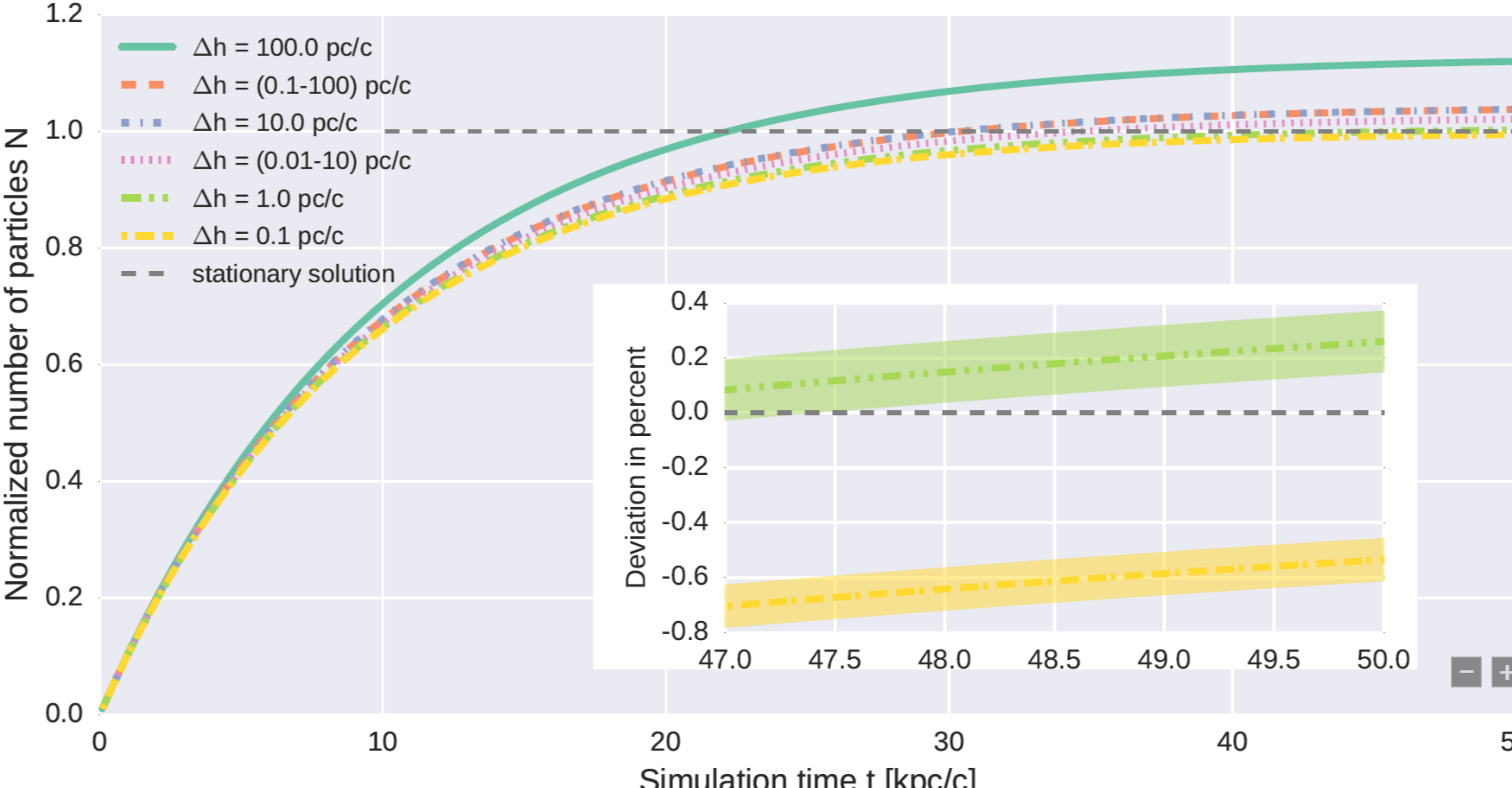


Fig 25. Total number density depending on maximum integration time for different integration time steps.

Validation IIa

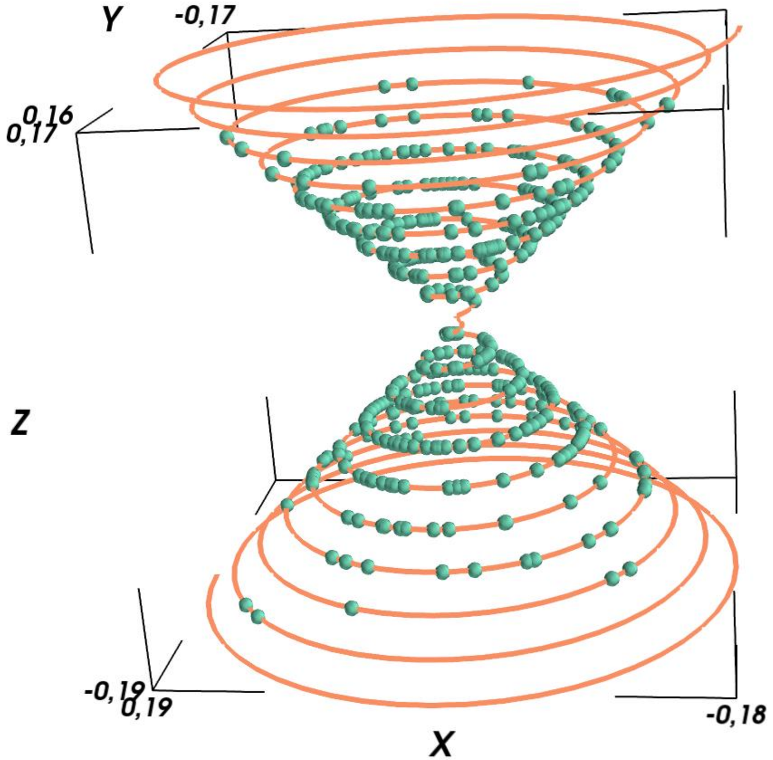


Fig 9: Example of a spiral field line and a sample of end positions.

- We test the accuracy of the algorithm in an artificial situation.
- A spiral with varying radius is used as the magnetic field line.
- The distance to the field line after the diffusion is taken as a measure for the algorithm accuracy.

Validation IIb

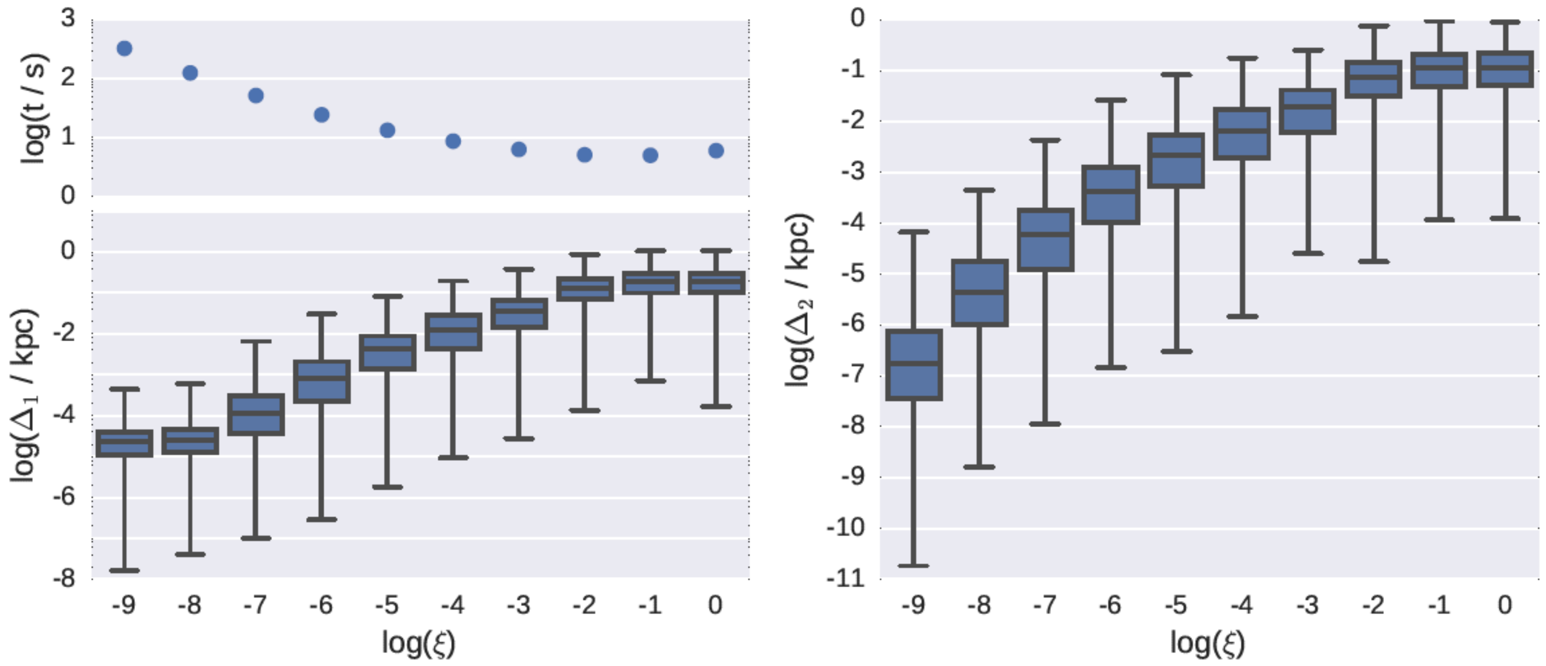


Fig 10: Results for the accuracy test. The algorithm allows a user chosen precision for a pure parallel diffusion.

Adiabatic Cooling

$$\frac{\partial n}{\partial t} + \vec{u} \cdot \nabla n = \nabla \cdot (\hat{\kappa} \nabla n) + \boxed{\frac{1}{3} (\nabla \cdot \vec{u}) \frac{\partial n}{\partial \ln p}} + S(\vec{x}, p, t)$$

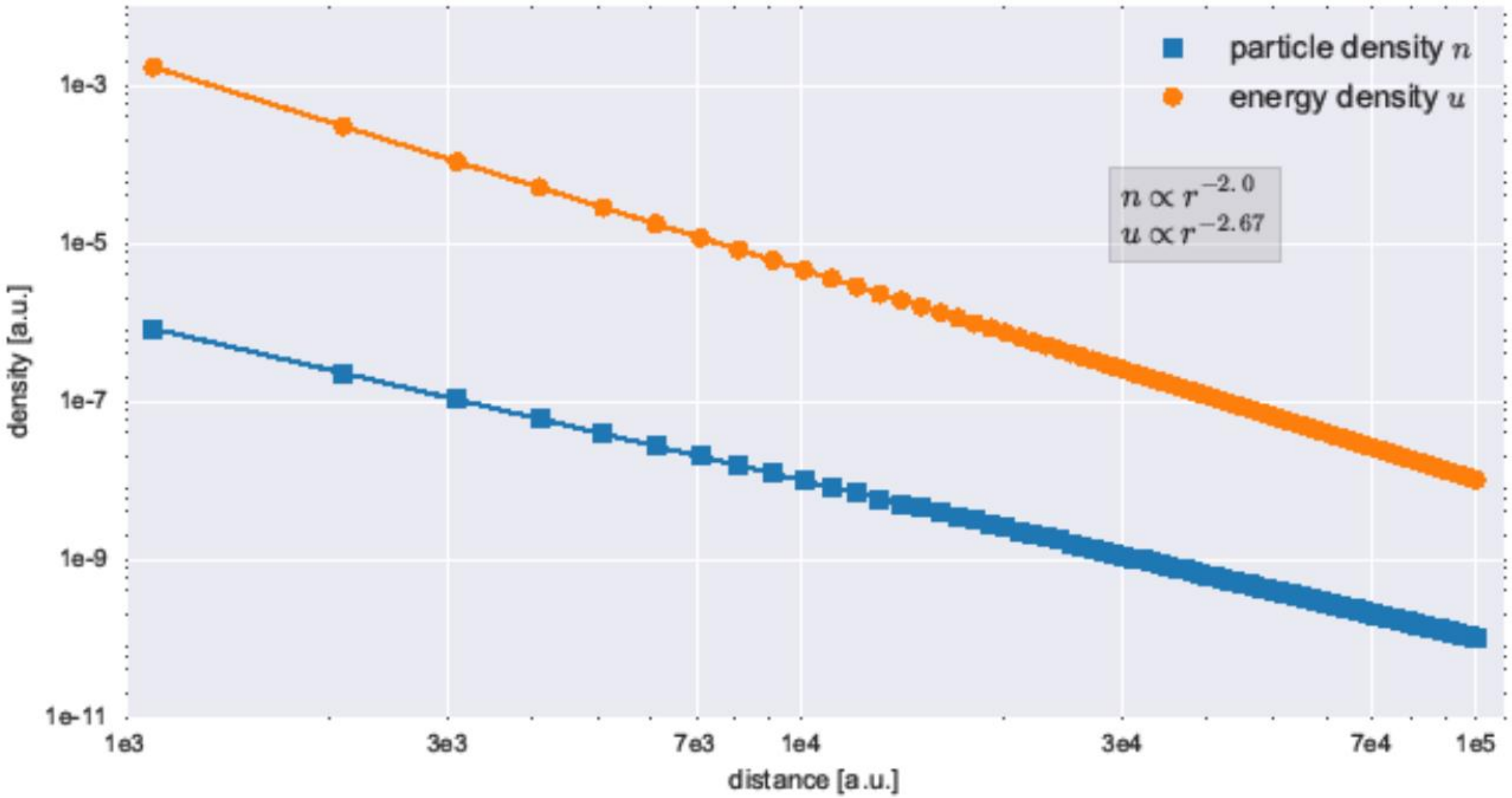


Fig 4. Particle and energy density for advective test case.

EXAMPLES

First applications – Rigidity

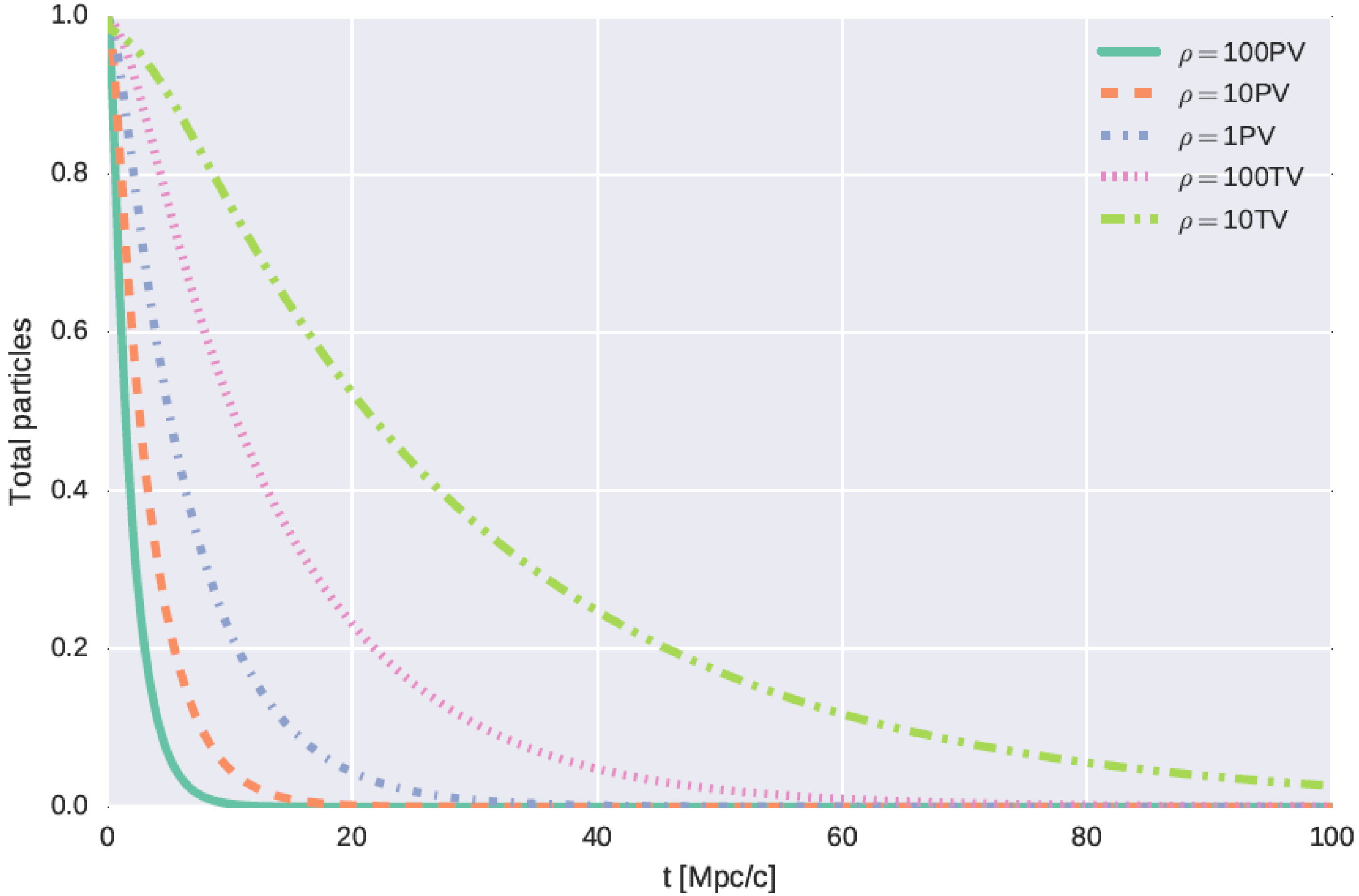


Fig 14: Time Evolution of the total particle number.

Continuous source

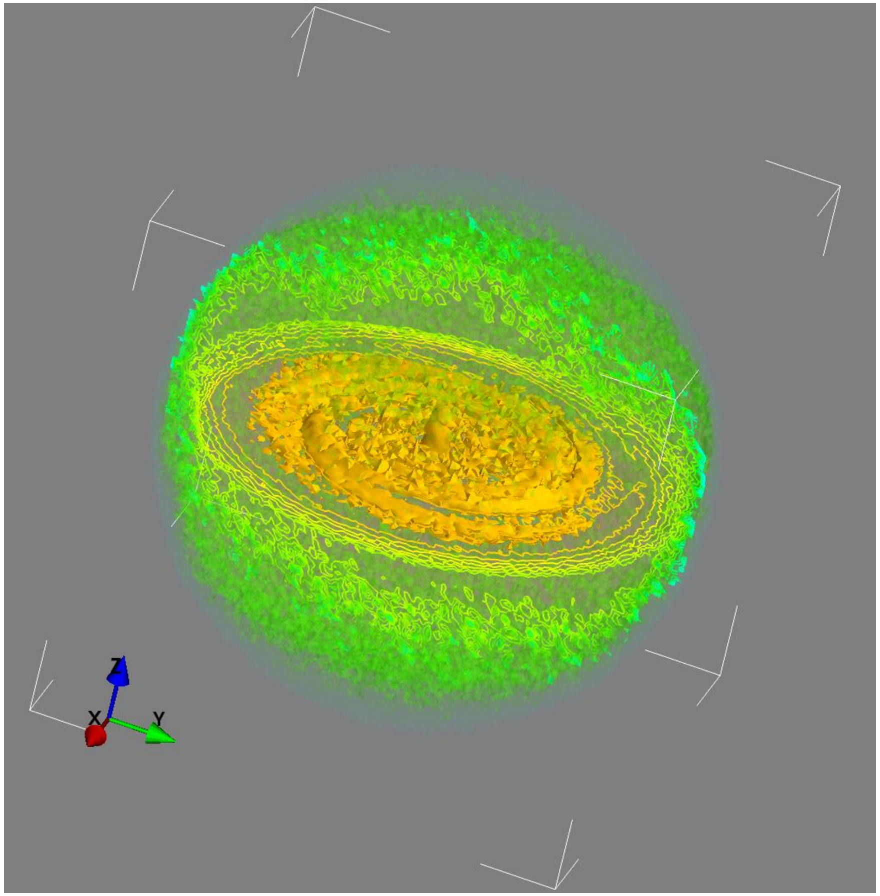
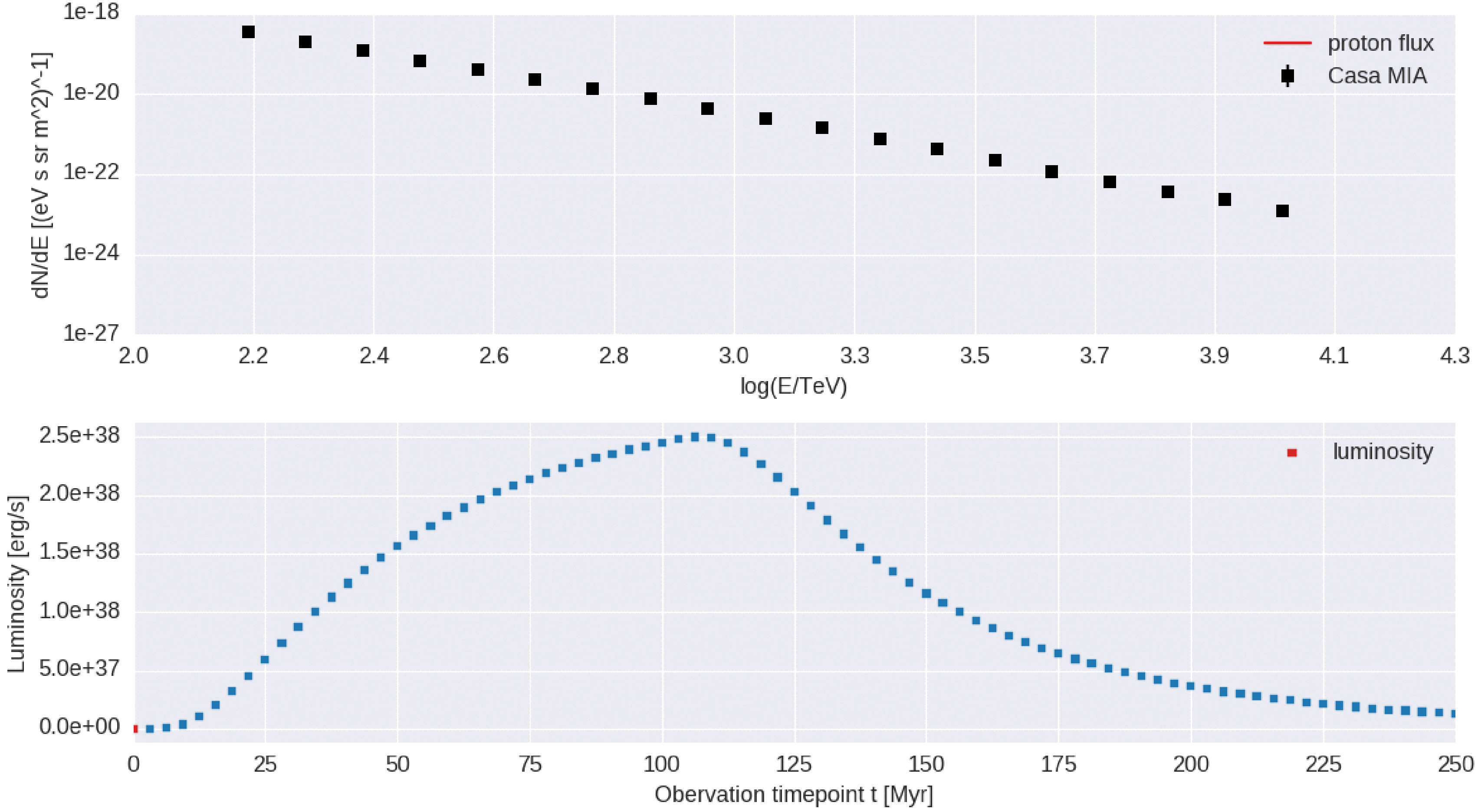
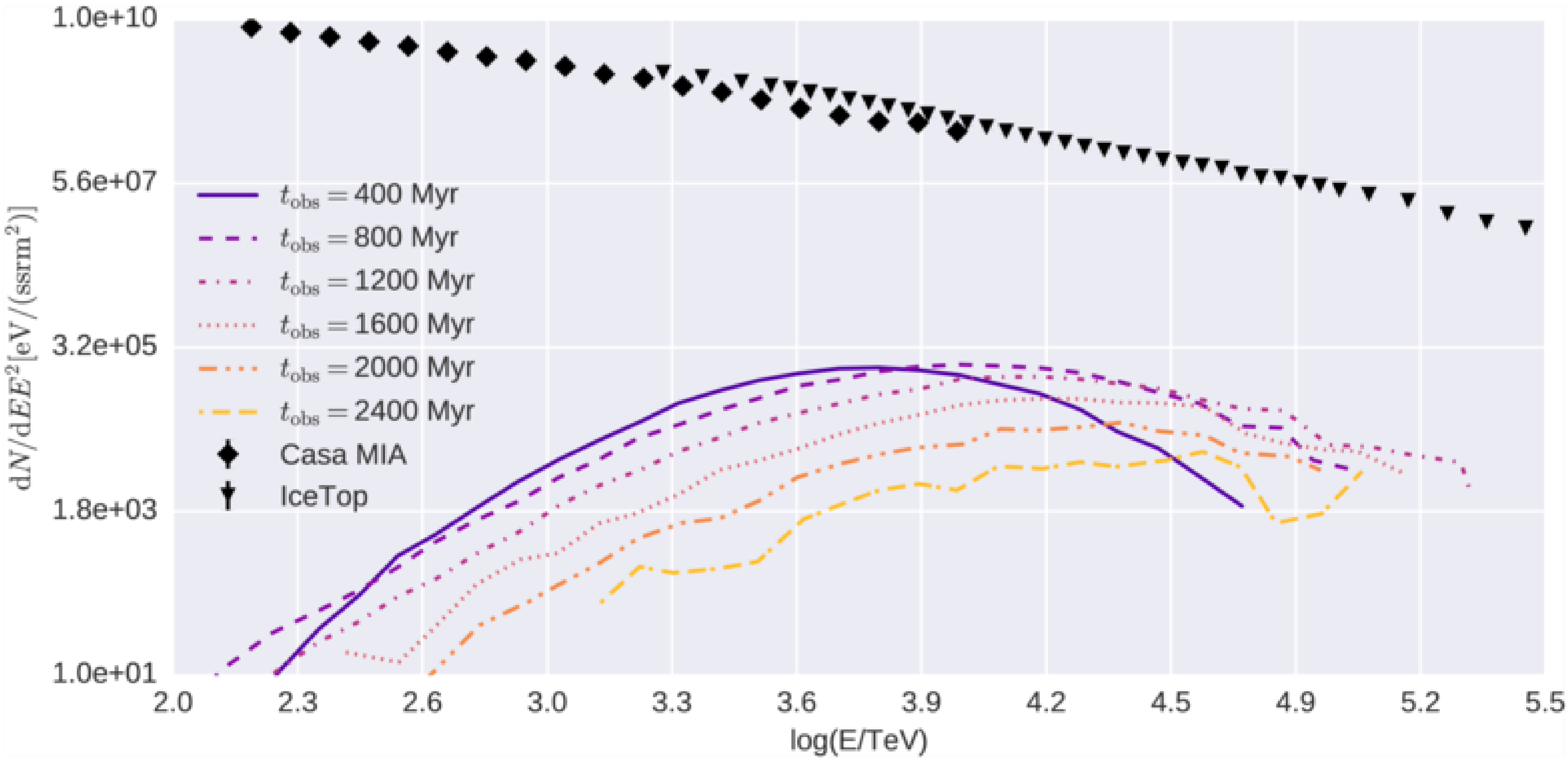


Fig 24: Cosmic ray density for continuous uniform emission inside the Galactic disc.

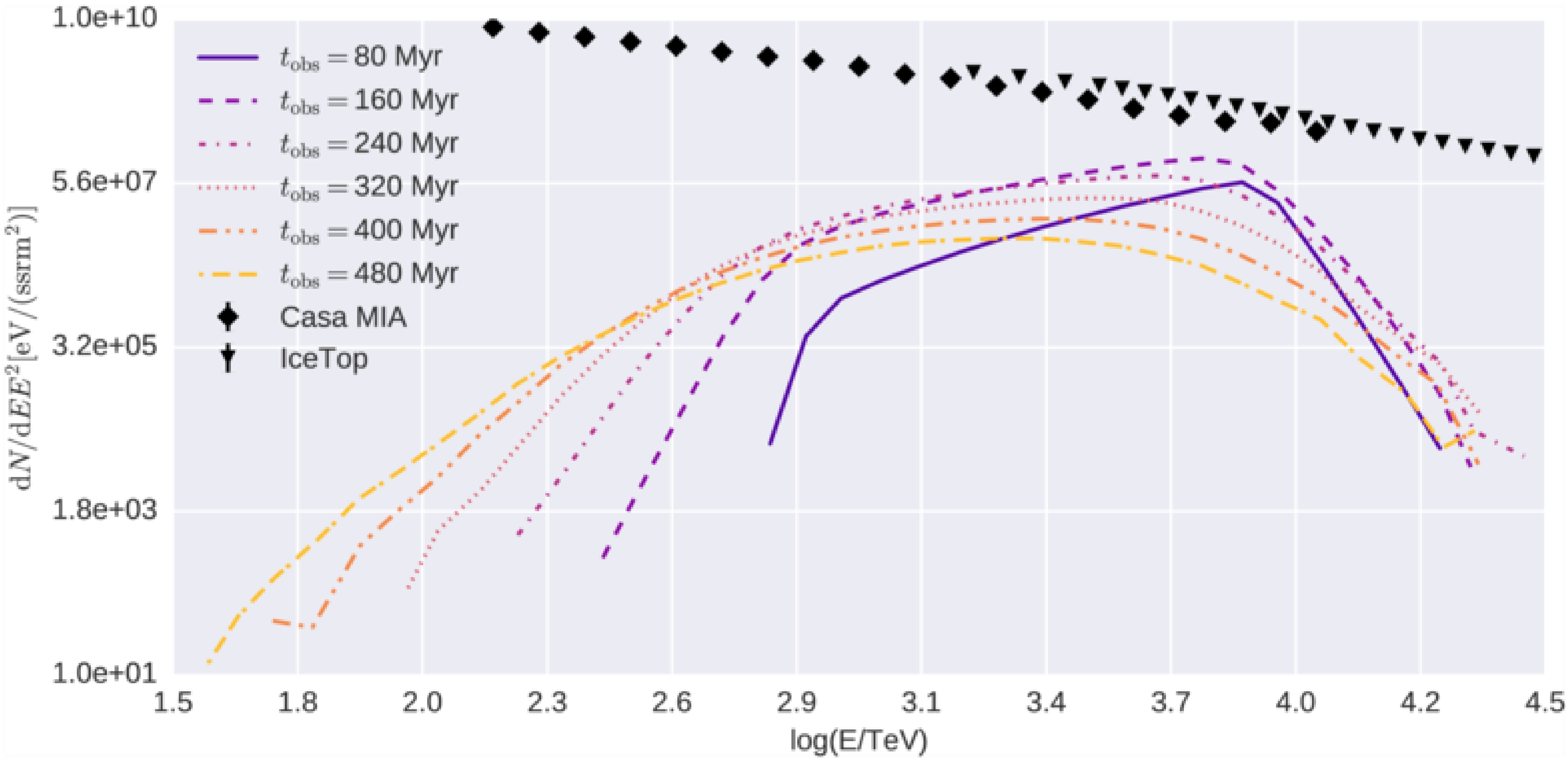
4 Time Evolution $\Delta t = 100$ Myr; $\delta = 0.5$



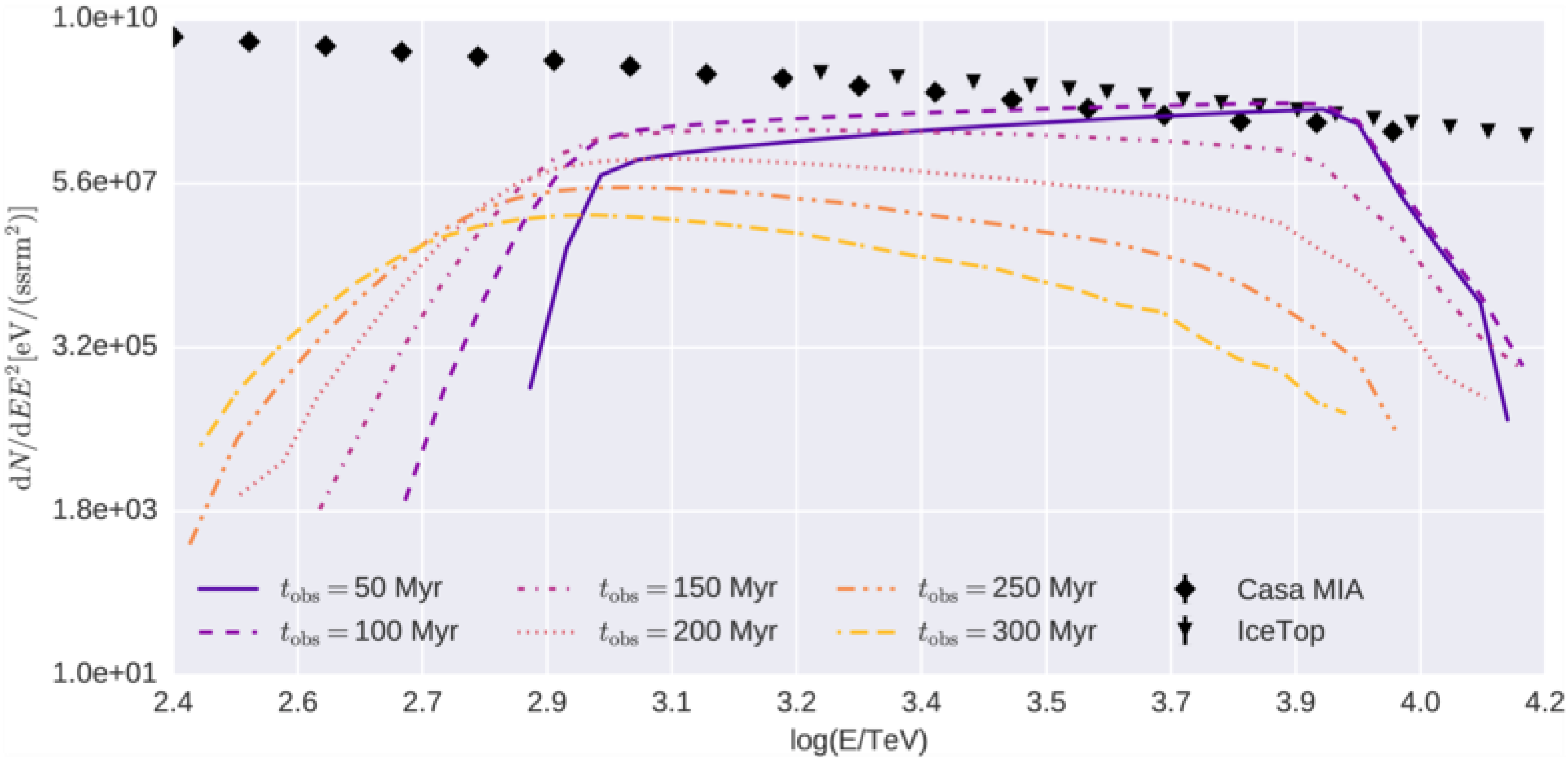
Time Evolution $\Delta t = 100$ Myr; $\delta = 0.3$



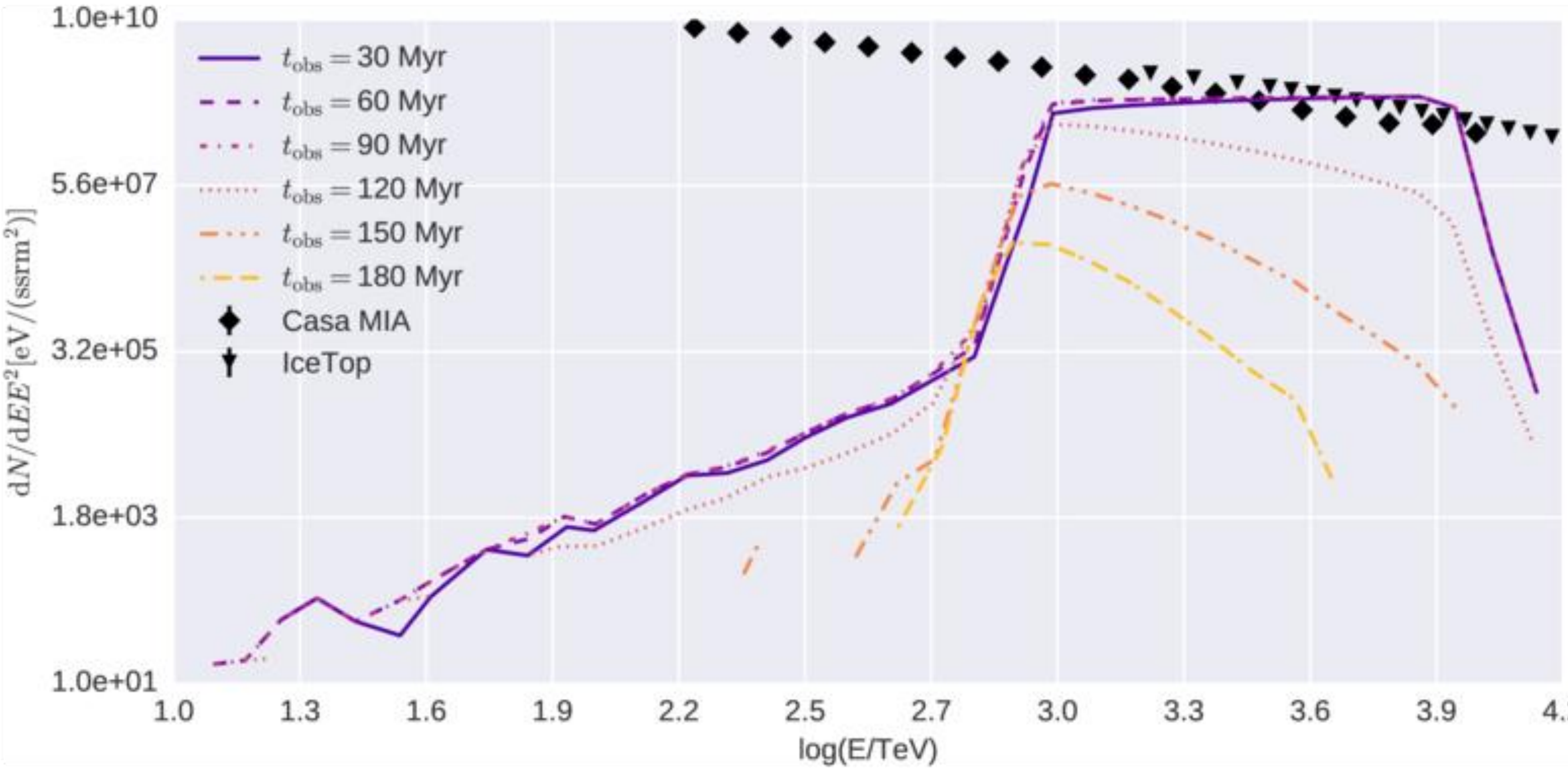
4 Time Evolution $\Delta t = 100$ Myr; $\delta = 0.4$



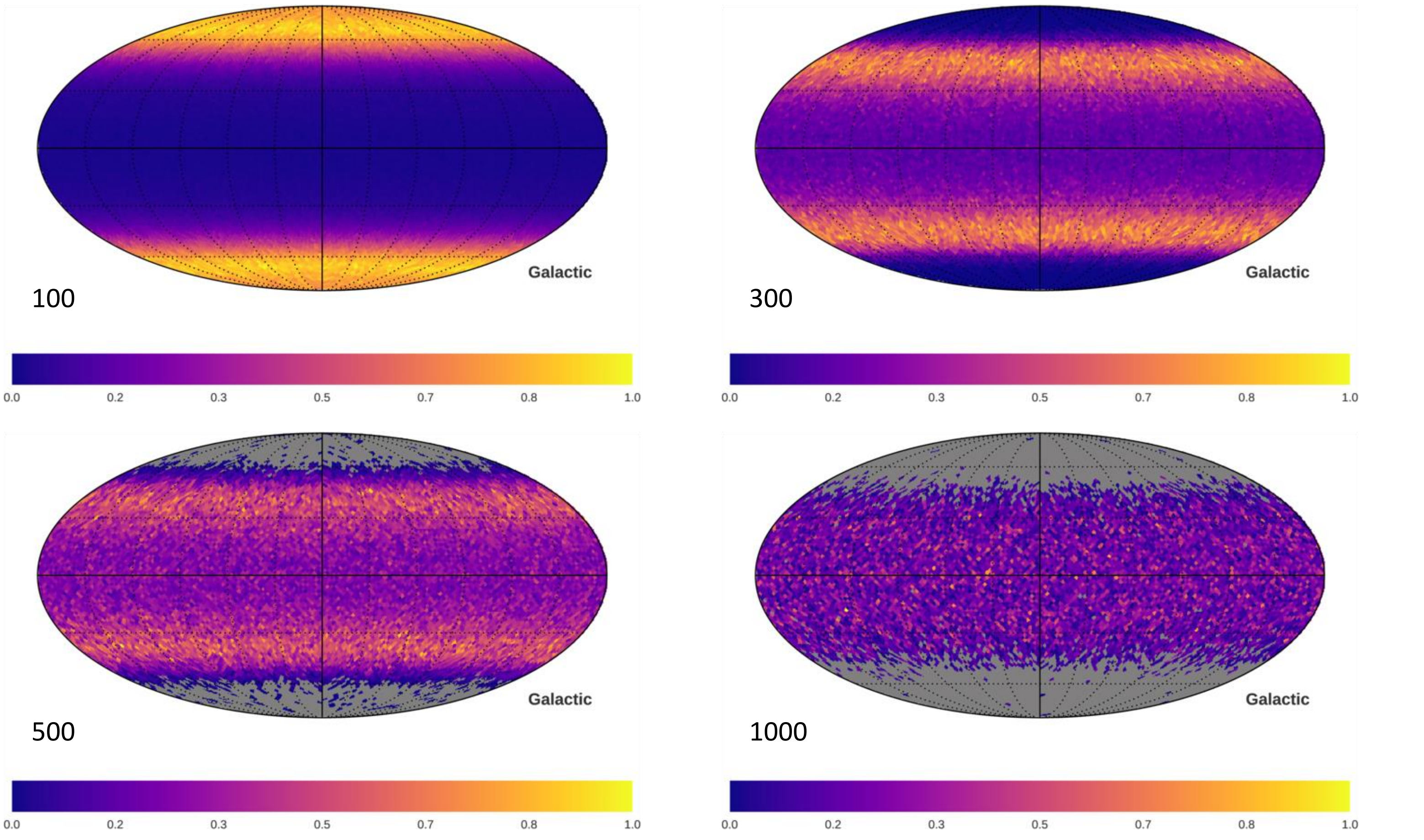
4 Time Evolution $\Delta t = 100$ Myr; $\delta = 0.5$



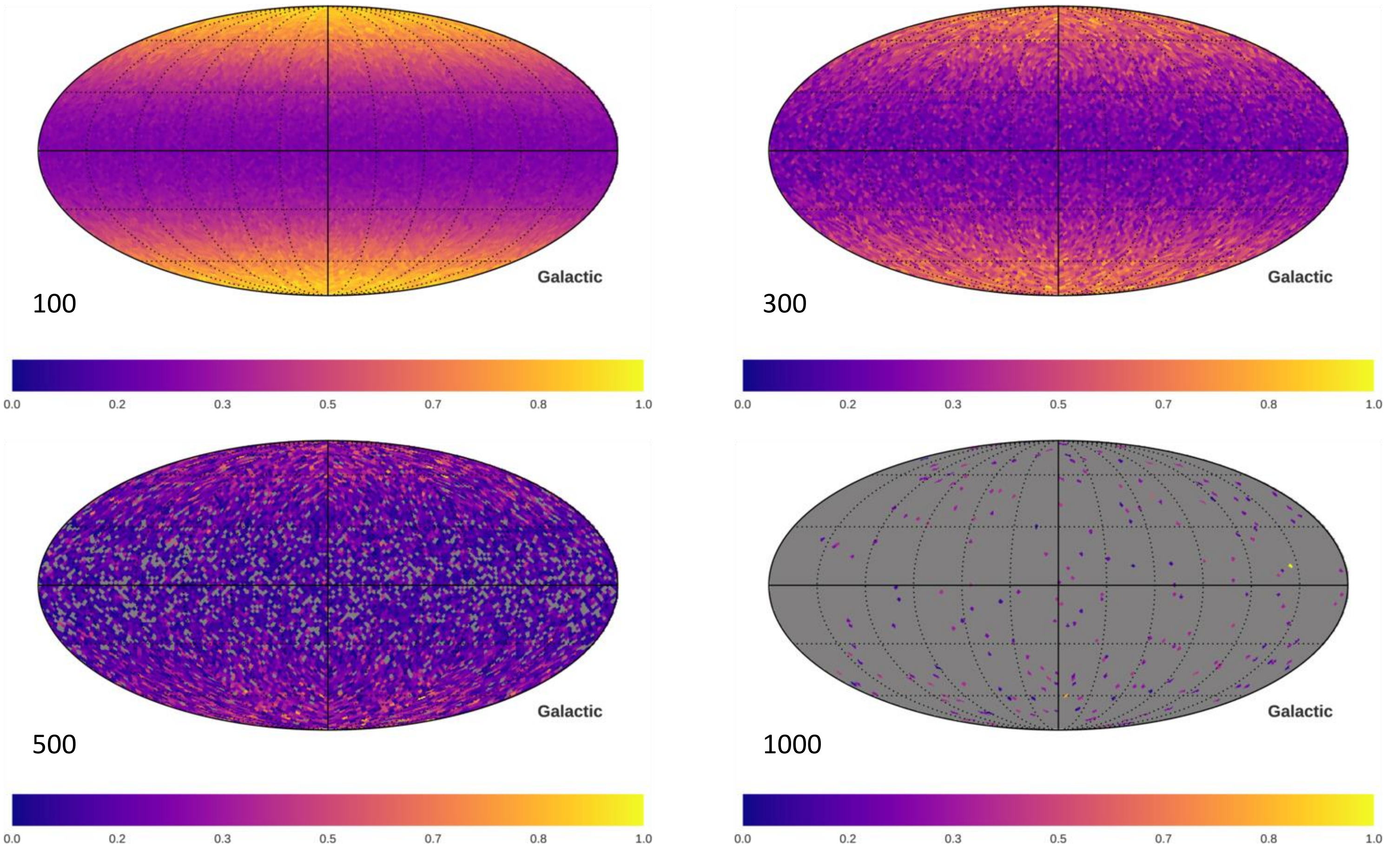
Time Evolution $\Delta t = 100$ Myr; $\delta = 0.6$



4 Arrival $\delta = 0.6, \epsilon = 0.$; Wind



4 Arrival $\delta = 0.6, \epsilon = 0.1$; Wind



EXTENSION

Outlook

Use the local turbulence ratio η with: $\eta = \dfrac{b_0^2}{b_0^2 + B_0^2}$ to calculate diffusion tensor.

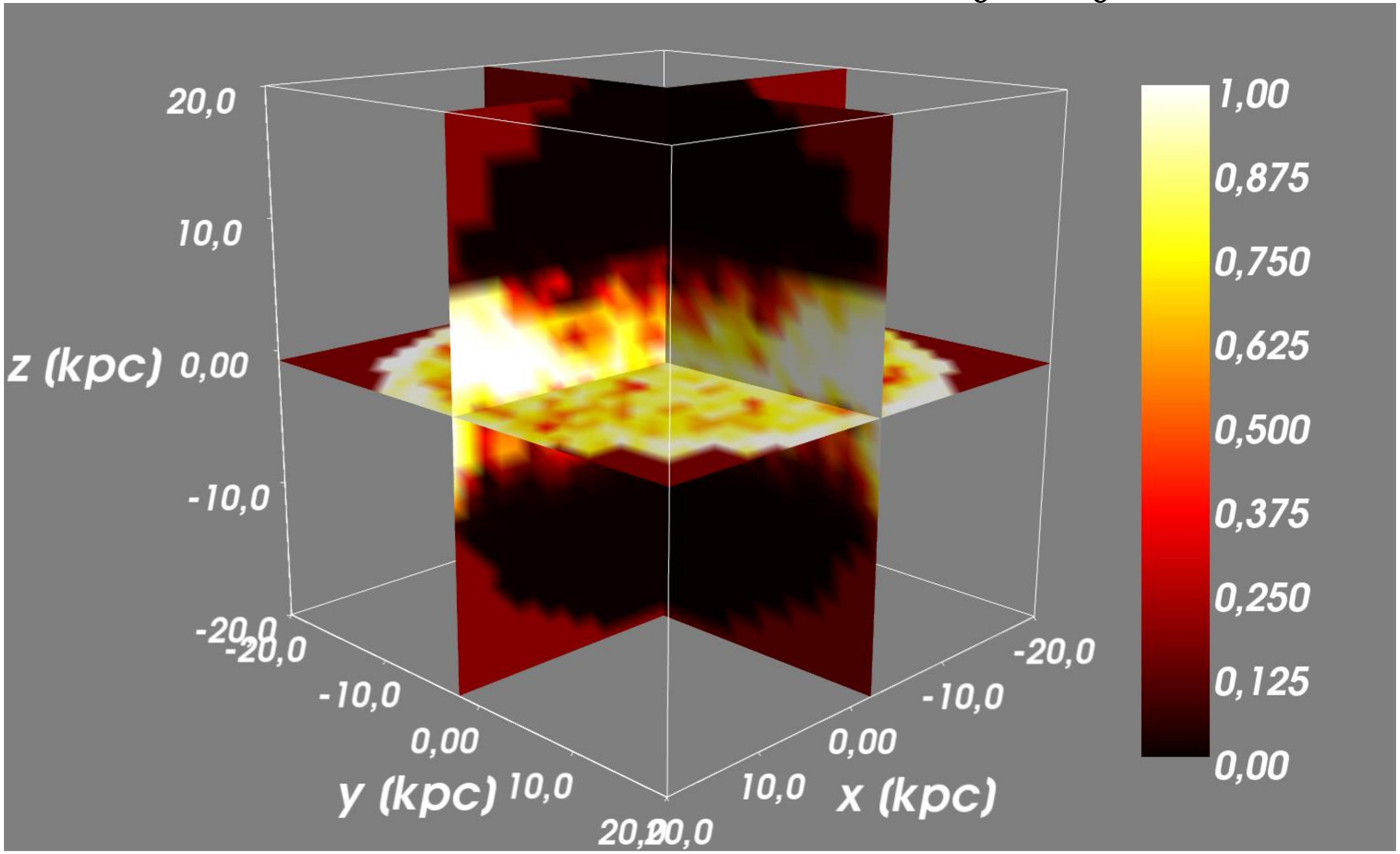


Fig 23: The turbulence ratio of the JF12 field.

COMPETITORS

Comparison of Tools

Tab. 1: Popular Propagation Programs

Name	Propa-gation	Diffusion	Integration	Inter-action	Remarks	Cite
GALPROP	Trans. Equ.	Scalar	Grid (Crank Nicolson)	Yes	Quasi stand.	Strong et al. (2011)
DRAGON 2	Trans. Equ.	3dim anisotr.	Grid	Yes		Evoli et al. (2016)
PICARD	Trans. Equ.	3dim anisotr.	Grid	Yes	Dedicated stat. Solver	Kissmann et al. (2014)
CRPropa 3 (PropagationCK)	Equ. of Motion	No	Cash Karp	Partly	UHECR	Batista et al. (2016)
CRPropa 3.1 (DiffusionSDE)	**Trans. Equ.**	**3dim const. Eigenvalues**	**SDE adaptive**	**Partly**	**Arbitrary magn. field**	**Merten et al. (t.b.s.)**
	Trans. Equ.	Fully anisotropic	SDE Euler-Mayurama	No		Kopp et al. (2011)