RUHR-UNIVERSITÄT BOCHUM

Lecture 2: Electromagnetic fluctuations and MHD eigenmodes

Reinhard Schlickeiser
Institut für Theoretische Physik IV: Weltraum- und Astrophysik, Ruhr-Universität Bochum, Germany

Erice 2018-08

Collaborators and topics

Main current collaborators: Ulrich Kolberg , Marian Lazar (RUB), Sunjung Kim (UNIST, Ulsan, Korea), Rodrigo Lopez, Peter H. Yoon (University of Maryland)

Topics:

1. Collective processes in plasmas
2. Linear fluctuations
3. Quasilinear transport equation
4. Non-collective fluctuation spectra
5. Collective eigenmodes
6. Slab MHD eigenmodes
7. Summary and conclusions

RUB

Collective processes in plasmas

Klimontovich particle density of sort "a":

$$N_a(\vec{x}, \vec{p}, t) = \sum_{i=1}^{N_0} \delta[\vec{x} - \vec{x}_i^a(t)]\delta[\vec{p} - \vec{p}_i^a(t)],$$

$$\dot{\vec{x}}_i^a = \vec{v}_i^a = \frac{\vec{p}_i^a}{m_a \gamma_i^a}, \quad \dot{\vec{p}}_i^a = q_a[\vec{E}(\vec{x}_i^a, t) + \frac{\vec{v}_i^a \times \vec{B}(\vec{x}_i^a, t)}{c}] \tag{1}$$

fulfils exact Klimontovich equation for each species "a":

$$\frac{\partial N_a}{\partial t} + \vec{v} \cdot \frac{\partial N_a}{\partial \vec{x}} + q_a \left[\vec{E} + \frac{\vec{v} \times \vec{B}}{c} \right] \cdot \frac{\partial N_a}{\partial \vec{p}} = 0 \tag{2}$$

The electromagnetic fields have to determined from the Maxwell equations

Collective processes (2)

$$\mathrm{div}\,\vec{E} = 4\pi \sum_a q_a \int d^3p\, N_a(\vec{x},\vec{p},t), \quad \mathrm{rot}\,\vec{E} + \frac{\partial \vec{B}}{c\partial t} = 0$$

$$\mathrm{div}\,\vec{B} = 0, \quad \mathrm{rot}\,\vec{B} - \frac{\partial \vec{E}}{c\partial t} = \frac{4\pi}{c} \sum_a q_a \int d^3p\, \vec{v} N_a(\vec{x},\vec{p},t) \tag{3}$$

Formal reduction (details in Yoon, Ziebell, Kontar, RS 2016, PR E 93, 033203; RS and Yoon 2015, Phys. Plasmas 22, 072108) in magnetized plasma ($\vec{B} = \vec{B}_0 + \delta\vec{B}$, $\vec{E} = \delta\vec{E}$) by introducing the free-particle (without turbulent fields) Klimontovich equation

$$\left[\frac{\partial}{\partial t} + \vec{v}\cdot\frac{\partial}{\partial \vec{x}} + \frac{q_a(\vec{v}\times\vec{B}_0)}{c}\cdot\frac{\partial}{\partial \vec{p}}\right] N_a^0(\vec{x},\vec{p},t)$$

$$= \mathcal{L}_0 N_a^0(\vec{x},\vec{p},t) = \left[\frac{\partial}{\partial t} + \vec{v}\cdot\frac{\partial}{\partial \vec{x}} - \Omega_a\frac{\partial}{\partial \phi}\right] N_a^0(\vec{x},\vec{p},t) = 0 \tag{4}$$

RUB

Collective processes (3)

with relativistic gyrofrequency $\Omega_a = q_a B_0/(\gamma m_a c)$ for free particle density

$$N_a^0(\vec{x}, \vec{p}, t) = \sum_{i=1}^{N_0} \delta[\vec{x} - \vec{x}_i^a]\delta[\vec{p} - \vec{p}_i^a] \tag{5}$$

with gyromotion in the uniform magnetic field

$$\begin{aligned} \vec{p}_i^a &= \vec{p}_{i,0}^a + (p_\perp \cos(\phi + \Omega_a t), p_\perp \sin(\phi + \Omega_a t), p_\parallel), \\ \vec{x}_i^a &= \vec{x}_{i,0}^a + (\frac{v_\perp}{\Omega_a}[\cos(\phi + \Omega_a t) - \cos\phi], \frac{v_\perp}{\Omega_a}[\sin(\phi + \Omega_a t) - \sin\phi], v_\parallel t) \end{aligned} \tag{6}$$

and non-turbulent time-propagation operator

$$\mathcal{L}_0 = \frac{\partial}{\partial t} + \vec{v} \cdot \frac{\partial}{\partial \vec{x}} - \Omega_a \frac{\partial}{\partial \phi} \tag{7}$$

in cylindrical momentum coordinates $p_x = p_\perp \cos\phi$, $p_y = p_\perp \sin\phi$, $p_z = p_\parallel$.

Collective processes (4)

Subtraction of Eq. (4) from Eq. (2) provides

$$\mathcal{L}_0[N_a(\vec{x},\vec{p},t) - N_a^0(\vec{x},\vec{p},t)] + q_a \left[\delta\vec{E} + \frac{\vec{v}\times\delta\vec{B}}{c}\right] \cdot \frac{\partial N_a(\vec{x},\vec{p},t)}{\partial \vec{p}} = 0 \quad (8)$$

This equation describes collective processes where initial purely single particle dynamics in the uniform magnetic field have been taken out.

Then split total microscopic particles quantities into averages and fluctuations:

$$N_a(\vec{x},\vec{p},t) = \langle N_a(\vec{x},\vec{p},t)\rangle + \delta N_a(\vec{x},\vec{p},t) \equiv f_a(\vec{x},\vec{p},t) + \delta N_a(\vec{x},\vec{p},t) \quad (9)$$

where ensemble-averages $\langle \delta N_a\rangle = \langle \delta E\rangle = \langle \delta B\rangle = 0$ of the fluctuations are zero, and $f_a(\vec{x},\vec{p},t) = \langle N_a(\vec{x},\vec{p},t)\rangle$ is the smoothed one-particle distribution function. Eq. (8) then reads

Collective processes (5)

$$\mathcal{L}_0[\langle N_a\rangle + \delta N_a - N_a^0] + q_a[\delta\vec{E} + \frac{\vec{v}\times\delta\vec{B}}{c}]\cdot[\frac{\partial\langle N_a\rangle}{\partial\vec{p}} + \frac{\partial\delta N_a}{\partial\vec{p}}] = 0 \qquad (10)$$

Ensemble-averaging this equation provides

$$\mathcal{L}_0 f_a = -q_a\langle(\delta\vec{E} + \frac{\vec{v}\times\delta\vec{B}}{c})\cdot\frac{\partial\delta N_a}{\partial\vec{p}}\rangle, \qquad (11)$$

so that the difference between Eqs. (10) and (11) is given by

$$\mathcal{L}_0[\delta N_a - N_a^0] + q_a(\delta\vec{E} + \frac{\vec{v}\times\delta\vec{B}}{c})\cdot\frac{\partial f_a}{\partial\vec{p}} =$$

$$-q_a\left[(\delta\vec{E} + \frac{\vec{v}\times\delta\vec{B}}{c})\cdot\frac{\partial\delta N_a}{\partial\vec{p}} - \langle(\delta\vec{E} + \frac{\vec{v}\times\delta\vec{B}}{c})\cdot\frac{\partial\delta N_a}{\partial\vec{p}}\rangle\right] \qquad (12)$$

Eqs. (9), (11) and (12) are still exact.

Linear fluctuations

Now we investigate small deviations $\delta\vec{B} \ll B_0$, $|\delta N_a| \ll f_a = n_a F_a$ (weak turbulence assumption) from the adopted "equilibrium" state of a spatially uniform and stationary plasma $(F_a(\vec{p}))$ in an uniform magnetic field $(\vec{B}_0 = B_0 \vec{e}_z)$. This leads to the linearized kinetic equation

$$\mathcal{L}_0[\delta N_a - N_a^0] + q_a(\delta\vec{E} + \frac{\vec{v} \times \delta\vec{B}}{c}) \cdot \frac{\partial f_a}{\partial \vec{p}} \simeq 0, \tag{13}$$

where we ignored the right-hand side of Eq. (12), being of 2-nd order in fluctuating quantities, as compared to its left-hand side, being of 1-st order in fluctuating quantities.

Eq. (13) guarantees that for $t = t^{'} = 0$ the two-time correlation function

$$\begin{aligned}\langle \delta N_a(\vec{x}, \vec{p}, 0) \delta N_a(\vec{x}^{'}, \vec{p}^{'}, 0) \rangle &= \langle N_a^0(\vec{x}, \vec{p}, 0) N_a^0(\vec{x}^{'}, \vec{p}^{'}, 0) \rangle \\ &= \delta[\vec{x} - \vec{x}^{'}] \delta[\vec{p} - \vec{p}^{'}] f_a(\vec{x}, \vec{p}, 0)\end{aligned} \tag{14}$$

Linear fluctuations (2)

equals the correlation function for uncorrelated free particles, which is obtained from the definition (5) as

$$\langle N_a^0(\vec{x},\vec{p},t) N_b^0(\vec{x}^{'},\vec{p}^{'},t^{'})\rangle = \delta_{ab}\delta[\vec{x}-\vec{x}^{'}]\delta[\vec{p}-\vec{p}^{'}]f_a(\vec{x},\vec{p},t) \tag{15}$$

In terms of the inverted time-integration (along the gyroorbit) operator $\mathcal{L}_0^{-1}$ the formal solution of the linearized kinetic equation (13) reads

$$\delta N_a = N_a^0 - q_a \mathcal{L}_0^{-1}(\delta\vec{E} + \frac{\vec{v}\times\delta\vec{B}}{c})\cdot\frac{\partial f_a}{\partial\vec{p}} \tag{16}$$

Upon insertion into Eq. (11) it provides

Quasilinear transport equation

the quasilinear kinetic equation for the ensemble-averaged phase space density

$$\mathcal{L}_0 f_a = \underbrace{-q_a \langle (\delta\vec{E} + \frac{\vec{v}\times\delta\vec{B}}{c}) \cdot \frac{\partial N_a^0}{\partial\vec{p}} \rangle}_{\text{drag terms}}$$

$$+ \underbrace{q_a^2 \langle (\delta\vec{E} + \frac{\vec{v}\times\delta\vec{B}}{c}) \cdot \frac{\partial}{\partial\vec{p}} \mathcal{L}_0^{-1} (\delta\vec{E} + \frac{\vec{v}\times\delta\vec{B}}{c}) \cdot \frac{\partial f_a}{\partial\vec{p}} \rangle}_{\text{generalized momentum diffusion}}, \qquad (17)$$

containing drag and generalized momentum diffusion terms. As the linear fluctuation theory below provides $(\delta E_i, \delta B_i) \propto N_b^0$ the drag terms depend on the correlation function (15) of uncorrelated particles and thus on $\partial f_a / \partial\vec{p}$.

Linearized Klimontovich-Maxwell equations

The linearized Klimontovich-Maxwell equations read

$$\nabla\cdot\delta\vec{E}(\vec{x},t)=4\pi\sum_a q_a\int d^3p\,\delta N_a(\vec{x},\vec{p},t),\ \nabla\times\delta\vec{E}(\vec{x},t)=-\frac{\partial}{c\partial t}\delta\vec{B}(\vec{x},t),$$

$$\nabla\cdot\delta\vec{B}(\vec{x},t)=0,\ \nabla\times\delta\vec{B}(\vec{x},t)-\frac{\partial}{c\partial t}\delta\vec{E}(\vec{x},t)=\frac{4\pi}{c}\sum_a q_a\int d^3p\ \vec{v}\,\delta N_a(\vec{x},\vec{p},t) \tag{18}$$

and

$$[\frac{\partial}{\partial t}+\ \vec{v}\cdot\frac{\partial}{\partial\vec{x}}-\Omega_a\frac{\partial}{\partial\phi}]\delta A_a(\vec{x},\vec{p},t)=-q_a n_a[\delta\vec{E}+\frac{\vec{v}\times\delta\vec{B}}{c}]\cdot\frac{\partial F_a(\vec{p})}{\partial\vec{p}},$$

$$\delta A_a(\vec{x},\vec{p},t)=\delta N_a(\vec{x},\vec{p},t)-N_a^0(\vec{x},\vec{p},t) \tag{19}$$

For $N_a^0(\vec{x},\vec{p},t)=0$ Eq. (19) reduces to the linearized Vlasov equation which cannot describe spontaneous emission of plasma fluctuations.

RUB

Gyrotropic equilibrium state

The adopted "equilibrium" state of a spatially uniform and stationary plasma ($F_a(\vec{p})$) fulfills the corresponding Maxwell and Klimontovich equations

$$0 = 4\pi \sum_a q_a n_a \int d^3p\, F_a(\vec{p}) = 4\pi \sum_a q_a n_a,$$

$$0 = \frac{4\pi}{c} \sum_a \vec{J}_a, \quad \vec{J}_a = q_a n_a \int d^3p\, \vec{v}\, F_a(\vec{p}),$$

$$q_a n_a \frac{\vec{v} \times \vec{B}_0}{c} \cdot \frac{\partial F_a(\vec{p})}{\partial \vec{p}} = -\Omega_a n_a \frac{\partial F_a(\vec{p})}{\partial \phi} = 0, \qquad (20)$$

Therefore our adopted equilibrium state demands gyrotropic distribution functions $F_a(\vec{p}) = F(p_{\|}, p_{\perp})$ independent of the gyrophase ϕ (caveat: unmagnetized plasmas).

Linear fluctuations (3)

We introduce the Fourier-Laplace transforms of the fluctuations in infinite space (no finite spatial boundaries)

$$\begin{pmatrix} A_a(\vec{k},\vec{p},\omega) \\ \vec{B}(\vec{k},\omega) \\ \vec{E}(\vec{k},\omega) \end{pmatrix} = \int_{-\infty}^{\infty} d^3x \int_0^{\infty} dt\, e^{-\imath\left(\vec{k}\cdot\vec{x}-\omega t\right)} \begin{pmatrix} \delta A_a(\vec{x},\vec{p},t) \\ \delta\vec{B}(\vec{x},t) \\ \delta\vec{E}(\vec{x},t) \end{pmatrix}, \qquad (21)$$

and their inverse transforms

$$(2\pi)^4 \begin{pmatrix} \delta A_a(\vec{x},\vec{p},t) \\ \delta\vec{B}(\vec{x},t) \\ \delta\vec{E}(\vec{x},t) \end{pmatrix} = \int_{-\infty}^{\infty} d^3k \int_L d\omega\, e^{\imath\left(\vec{k}\cdot\vec{x}-\omega t\right)} \begin{pmatrix} A_a(\vec{k},\vec{p},\omega) \\ \vec{B}(\vec{k},\omega) \\ \vec{E}(\vec{k},\omega) \end{pmatrix} \qquad (22)$$

with the real wave vector $\vec{k}$ and the complex frequency $\omega = \omega_R + \imath\Gamma$. The imaginary part of $\Im\omega = \Gamma > 0$ is chosen sufficiently positive so that the Laplace integral (21) converges (note $e^{\imath\omega t} = e^{\imath\omega_R t - \Gamma t}$).

Linear fluctuations (4)

1) Reasoning for $\Gamma > 0$: multiply Eqs. (18) - (19) by $e^{-\imath\left(\vec{k}\cdot\vec{x}-\omega t\right)}$ and integrate over time as in (21). This leads e.g. to

$$\int_0^\infty dt\, \frac{\partial \delta B(\vec{x},t)}{\partial t} e^{\imath \omega t} = [\delta B(\vec{x},t) e^{\imath \omega_R t - \Gamma t}]_0^\infty - \imath \omega \int_0^\infty dt\, \delta B(\vec{x},t) e^{\imath \omega t}$$

$$\to -\delta B(\vec{x},t=0) - \imath \omega \vec{B}(\vec{k},\omega), \tag{23}$$

only for $\Gamma > 0$ eliminating $\delta B(\vec{x}, t=\infty)$ from the partial integration.

2) The temporal dependence associated with the spectral amplitude, $A_a(\mathbf{k}, \omega, \vec{p}, t)$, in Eq. (22), is supposed to be weakly adiabatic. Strictly speaking, the amplitude must be independent of t, but we retain the adiabatic slow-time dependence for reasons to be explained later. This prescription is a shortcut approach to the more rigorous multiple-time scale analysis (Davidson 1972).

Linear fluctuations (5)

In the following we absorb the adiabatic time dependence into the definition for the generalized angular frequency, $\omega \rightarrow \omega + \imath(\partial/\partial t)$, and shall reintroduce the slow-time derivative later.

3) In order to derive damped fluctuations with negative $\Gamma < 0$ we have to analytically continue any complex function $\Lambda(\vec{k}, \omega_R, \Gamma)$ into the negative complex frequency plane (Landau 1946), so that

$$\lim_{\Gamma \to 0^-} \Lambda(\vec{k}, \omega_R, \Gamma) = \lim_{\Gamma \to 0^+} \Lambda(\vec{k}, \omega_R, \Gamma) \tag{24}$$

Wave equation

The Fourier-Laplace transformation of the linearized Maxwell equations (18) then read (initial fields taken care of by N_a^0 !)

$$\vec{k}\cdot\vec{E}(\vec{k},\omega) = -4\pi\imath\rho(\vec{k},\omega), \quad \rho(\vec{k},\omega) = \sum_a q_a \int d^3p\, N_a(\vec{k},\omega,\vec{p}),$$
$$\vec{B}(\vec{k},\omega) = \frac{c}{\omega}\vec{k}\times\vec{E}(\vec{k},\omega), \quad \vec{k}\cdot\vec{B}(\vec{k},\omega) = 0,$$
$$\vec{k}\times\vec{B}(\vec{k},\omega) + \frac{\omega}{c}\vec{E}(\vec{k},\omega) = -\frac{4\pi\imath}{c}\vec{j}(\vec{k},\omega),$$
$$\vec{j}(\vec{k},\omega) = \sum_a q_a \int d^3p\, \vec{v} N_a(\vec{k},\omega,\vec{p}) \qquad (25)$$

Eliminating $\vec{B}(\vec{k},\omega)$ provides the wave equation

$$\vec{k}\times(\vec{k}\times\vec{E}(\vec{k},\omega)) + \frac{\omega^2}{c^2}\vec{E}(\vec{k},\omega) = -\frac{4\pi\imath\omega}{c^2}\vec{j}(\vec{k},\omega), \qquad (26)$$

Wave equation (2)

where the fluctuating current density $\vec{j}(\vec{k},\omega)$ is calculated from the Fourier-Laplace-transformed linearized Klimontovich equation

$$\imath\left(\omega-\vec{k}\cdot\vec{v}\right)A_a(\vec{k},\omega,\vec{p})+\ \Omega_a\frac{\partial A_a(\vec{k},\omega,\vec{p})}{\partial\phi}$$

$$=q_a n_a\left[\vec{E}(\vec{k},\omega)+\frac{\vec{v}\times(\vec{k}\times\vec{E}(\vec{k},\omega))}{\omega}\right]\cdot\frac{\partial F_a(\vec{p})}{\partial\vec{p}} \quad (27)$$

with

$$A_a(\vec{k},\omega,\vec{p})=N_a(\vec{k},\omega,\vec{p})-N_a^0(\vec{k},\omega,\vec{p}) \quad (28)$$

For **unmagnetized** plasmas with $B_0=\Omega_a=0$ and $\vec{k}=k\vec{e}_z$ the linearized Klimontovich equation (27) simplifies to (see RS and Yoon 2015, Physics of Plasmas 22, 072108 for magnetized plasmas)

RUB

Unmagnetized plasma

$$A_a(\vec{k},\omega,\vec{p}) = \frac{q_a n_a}{\omega - \vec{k}\cdot\vec{v}} \left[\vec{E}(\vec{k},\omega) + \frac{\vec{v}\times(\vec{k}\times\vec{E}(\vec{k},\omega))}{\omega}\right]\cdot\frac{\partial F_a(\vec{p})}{\partial\vec{p}}, \qquad (29)$$

yielding from Eq. (28)

$$N_a = N_a^0 + \frac{q_a n_a}{\omega - \vec{k}\cdot\vec{v}} \left[\vec{E}(\vec{k},\omega) + \frac{\vec{v}\times(\vec{k}\times\vec{E}(\vec{k},\omega))}{\omega}\right]\cdot\frac{\partial F_a(\vec{p})}{\partial\vec{p}}, \qquad (30)$$

so that the linearized fluctuating plasma current density (25) consists of two parts

$$\vec{j} = \vec{j}_0(\vec{k},\omega) + \vec{j}_{\mathrm{ind}}(\vec{k},\omega), \;\; \vec{j}_0 = \sum_a e_a \int d^3p\, \vec{v} N_a^0(\vec{k},\omega,\vec{p}) \qquad (31)$$

Unmagnetized plasma (2)

where $\vec{j}_0$ is caused by $N_a^0(\vec{k},\omega,\vec{p})$, accounting for the near interactions of neighbouring uncorrelated charged particles in the plasma. This spontaneous current density gives rise to the spontaneous emission of electromagnetic fields.

On the other hand, with the plasma frequency $\omega_{p,a}^2 = 4\pi q_a^2 n_a/m_a$ of species a

$$\vec{j}_{\rm ind}(\vec{k},\omega) = \frac{1}{4\pi}\sum_a m_a\omega_{p,a}^2 \int d^3p\, \frac{\vec{v}}{\omega-\vec{k}\cdot\vec{v}}[\vec{E}(\vec{k},\omega)$$
$$+\frac{\vec{v}\times(\vec{k}\times\vec{E}(\vec{k},\omega))}{\omega}]\cdot\frac{\partial F_a(\vec{p})}{\partial\vec{p}} \propto \vec{E}, \quad j_{\rm ind,i} = \sigma_{ij}E_j \tag{32}$$

is induced by the plasma fluctuations, where σ_{ij} is the conductivity tensor.

Wave equation in unmagnetized plasma

In terms of field components the wave equation (26) then becomes

$$\Lambda_{ij}^{+}(\vec{k},\omega)E_j(\vec{k},\omega) = E_i^0(\vec{k},\omega), \quad E_i = (\Lambda_{in}^{+})^{-1}E_n^0 \tag{33}$$

with the Maxwell operator (also determining the polarization state)

$$\Lambda_{ij}^{+}(\vec{k},\omega) = \frac{k^2c^2}{\omega^2}\left(\frac{k_ik_j}{k^2} - \delta_{ij}\right) + \psi_{ij}^{+}(\vec{k},\omega), \quad \psi_{ij}^{+} = \delta_{ij} + \frac{4\pi\imath\sigma_{ij}^{+}}{\omega} \tag{34}$$

and

$$E_i^0(\vec{k},\omega) = -\frac{4\pi\imath}{\omega}\sum_a q_a \int d^3p\, v_i N_a^0(\vec{k},\omega,\vec{p}), \tag{35}$$

denoting the electric field perturbations caused by uncorrelated single source particles in the plasma.

The index $+$ indicates that the Maxwell operator has been derived initially for a positive imaginary part of the frequency $\Gamma > 0$. For negative $\Gamma < 0$ proper analytical continuation of the complex Maxwell operator is required.

Non-collective fluctuation spectra

From Eq. (33) we obtain the non-collective electric field fluctuation spectra

$$< E_i(\vec{k},\omega)E_j^*(\vec{k},\omega) >= \Lambda_{im}^{-1}(\vec{k},\omega)\Lambda_{jn}^{*-1}(\vec{k},\omega) < E_m^0(\vec{k},\omega)E_n^{0*}(\vec{k},\omega) >,$$

$$\Lambda_{im}^{-1}(\vec{k},\omega) = \frac{\lambda_{im}(\vec{k},\omega)}{\Lambda(\vec{k},\omega)}, \quad \Lambda(\vec{k},\omega) = \det\left(\Lambda_{ij}(\vec{k},\omega)\right), \tag{36}$$

in terms of the matrix cofactors λ_{im}. In unmagnetized plasmas (for magnetized plasmas more complicated see Yoon, Lopez, Vafin, Kim, RS, 2017, Plasma Phys. Contr. Fus. 59, 095002) $\Lambda = \Lambda_L \Lambda_T^2$ with

$$\Lambda_{ij} = \begin{pmatrix} \Lambda_T & 0 & 0 \\ 0 & \Lambda_T & 0 \\ 0 & 0 & \Lambda_L \end{pmatrix}, \quad \Lambda_{im}^{-1} = \frac{1}{\Lambda}\begin{pmatrix} \Lambda_T\Lambda_L & 0 & 0 \\ 0 & \Lambda_T\Lambda_L & 0 \\ 0 & 0 & \Lambda_T^2 \end{pmatrix} \tag{37}$$

with

Non-collective fluctuation spectra (2)

the transverse dispersion function

$$\Lambda_T = 1 - \frac{c^2 k^2}{\omega^2} + \sum_a \frac{\pi m_a \omega_{p,a}^2}{\omega^2} \int_{-\infty}^{\infty} dp_\parallel \int_0^\infty dp_\perp \frac{p_\perp^2}{\gamma} \left[\frac{\partial F_a}{\partial p_\perp} + \frac{k v_\perp}{\omega - k v_\parallel} \frac{\partial F_a}{\partial p_\parallel} \right] \tag{38}$$

and the longitudinal dispersion function

$$\Lambda_L = 1 + \frac{2\pi}{\omega} \sum_a m_a \omega_{p,a}^2 \int_{-\infty}^{\infty} dp_\parallel p_\parallel \int_0^\infty dp_\perp \frac{p_\perp}{\gamma(\omega - k v_\parallel)} \frac{\partial F_a}{\partial p_\parallel}, \tag{39}$$

valid for **any** gyrotropic equilibrium distribution function $F_a(p_\perp, p_\parallel)$.

As only non-vanishing elements we find

Non-collective fluctuation spectra (3)

RUB

$$< E_\perp^2 > (k,\omega) = \sum_a \frac{\omega_{p,a}^2 m_a}{2\pi^2 |\omega \Lambda_T|^2} \int_{-\infty}^{\infty} dp_\parallel \int_0^\infty dp_\perp p_\perp v_\perp^2 F_a(p_\parallel, p_\perp)$$

$$\times \frac{|\Gamma|}{(\omega_R - k_\parallel v_\parallel)^2 + \Gamma^2}, \quad < B_\perp^2 > (k,\omega) = \frac{c^2 k^2}{|\omega|^2} < E_\perp^2 > (k,\omega),$$

$$< E_\parallel^2 > (k,\omega) = \sum_a \frac{\omega_{p,a}^2 m_a}{2\pi^2 |\omega \Lambda_L|^2} \int_{-\infty}^{\infty} dp_\parallel \int_0^\infty dp_\perp p_\perp v_\parallel^2 F_a(p_\parallel, p_\perp)$$

$$\times \frac{|\Gamma|}{(\omega_R - k_\parallel v_\parallel)^2 + \Gamma^2}, \tag{40}$$

which are dominated by contributions from the collective plasma eigenmodes defined by $\Lambda_{T,L}(k,\omega) = 0$.

Results: 4th eigenmode of isotropic unmagnetized plasmas

RUB

The most prominent unmagnetized cosmic plasmas are the intergalactic medium (IGM) and the early universe with thermal distribution functions $F_a(p) = C_a e^{-\mu_a \gamma_a}$, $\mu_a = m_a c^2/(k_B T_0)$ and $\gamma_a = \sqrt{1 + (p/m_a c)^2}$.

In standard plasma physics textbooks it is stated that unmagnetized isotropic plasmas sustain only three eigenmodes: (1) transverse (with $\delta\vec{B} \perp \delta\vec{E} \perp \vec{k}$) superluminal electromagnetic waves,
(2) longitudinal (with $\delta\vec{B} \propto \vec{k} \times \delta\vec{E} = 0$ and $\delta\vec{E} \parallel \vec{k}$) subluminal electrostatic waves,
(3) in electron-ion-plasmas longitudinal ion sound waves.

Felten et al. (2013): 4th eigenmode exists in the form of damped ($\Gamma < 0$) transverse aperiodic ($\omega_R = 0$) fluctuations, which oscillate in space but do not propagate. It is the only subluminal eigenmode with magnetic field fluctuations. Felten and RS (2013): it exists in any isotropic unmagnetized plasma, irrespective of the explicit form of the momentum distribution function of plasma particles.

Thermal aperiodic noise in the pamyprepanne

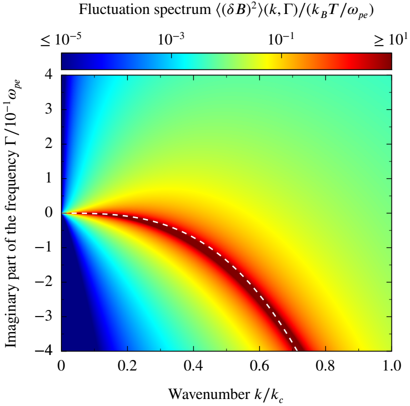

Figure 1: Contour plot of the spontaneously emitted thermal aperiodic magnetic noise at the end of the pamyprepanne for a temperature of $k_B T = 1$ MeV. The colour scale is logarithmic in powers of 10. The dashed line shows the dispersion relation of the damped eigenmode. From RS, Kolberg and Yoon (2018, ApJ 857, 29).

Thermal aperiodic noise in the pamyprepanne (2)

RUB

By integrating the fluctuation spectra over all wavenumber and frequency values:

$$|\delta B(t_0)| = \sqrt{(\delta B)^2(t_0)} = 2.2 \cdot 10^{12}\ \mathrm{G}, \quad |\delta E(t_0)| = 1.4 \cdot 10^{12}\ \mathrm{G} \tag{41}$$

The electric plus magnetic energy density in aperiodic fluctuations

$$w_{B+E}(t) = \frac{(\delta B)^2 + (\delta E)^2}{8\pi} = 2.7 \cdot 10^{23}\, T_{\mathrm{MeV}}^4\ \mathrm{erg\ cm}^{-3}, \tag{42}$$

which is about three orders of magnitude smaller than the photon energy density $w_p(t) = 1.4 \cdot 10^{26}\, T_{\mathrm{MeV}}^4$ erg cm^{-3}, so that the aperiodic fluctuations have a negligible influence on the radiation-driven cosmological evolution during the pamyprepanne.

Thermal aperiodic noise in the present-day IGM

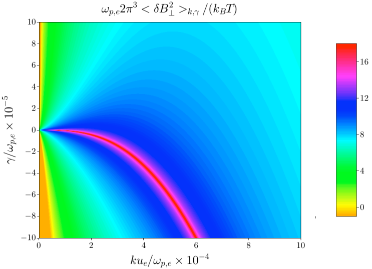

Figure 2: Spontaneously emitted thermal aperiodic magnetic noise in the IGM for $T_0 = 10^4$K and ionized gas densities of $n_e = 10^{-7}$ cm^{-3} . The colour scale is logarithmic in powers of e. From Felten, RS, Yoon and Lazar (2013, Phys. Plasmas 20, 052113).

Again the spectrum is dominated by a damped mode.

Thermal aperiodic noise in the IGM (2)

By integrating the fluctuation spectra over all wavenumber and frequencya values the random magnetic and electric field strengths in the form of aperiodic fluctuations in the present IGM are at $z = 0$

$$|\delta B(z=0)| = 6.3 \cdot 10^{-18}\, n_{-7}^{3/4}\, T_4^{1/8}\ \mathrm{G},$$
$$|\delta E(z=0)| = 1.9 \cdot 10^{-16} n_{-7}^{2/3}\, T_4^{1/2}\ \mathrm{G} \quad (43)$$

With $n_{-7}(z) = n_{-7}(1+z)^3$ and $T_4(z) = T_4(1+z)$ at reionization onset

$$|\delta B(z=20)| = 8.7 \cdot 10^{-15}\ \mathrm{G} \quad (44)$$

This guaranteed EGMF in the form of randomly distributed aperiodic oscillations with volume filling factor $\mathcal{V} \simeq 1$, serves as seed fields for possible amplification by later possible plasma instabilities from anisotropic plasma particle distribution functions, MHD instabilities and/or the MHD dynamo process (RS 2012, PRL 109, 261101).

Results for magnetized plasmas

Kim, Lazar, RS, Lopez, Yoon (2018, PPCF 60, 075010): weakly-damped ($\Gamma \ll \omega_R$) magnetic fluctuation spectra for isotropic Kappa-distribution with $\kappa = 2$:

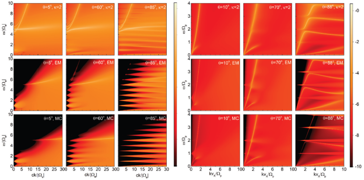

FIG. 2. Fluctuations from theory: high-frequency (left) and low-frequency (right).

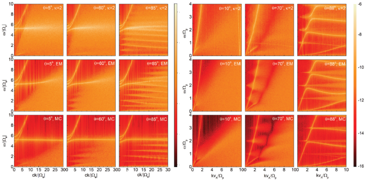

FIG. 3. Fluctuations from simulations: high-frequency (left) and low-frequency (right).

Collective eigenmodes

A collective plasma eigenmode with $\varpi_k = \omega_R(k) + \imath\Gamma_k$ has to fulfil the dispersion relation $\Lambda(k, \varpi_k) = 0$ with the dispersion functions (38) and (39). We recall e.g. for slab fluctuations (parallel ($\vec{k} \times \vec{B} = 0$) wave vector)

$$\Lambda_L - 1 = \frac{2\pi}{\omega} \sum_a m_a \omega_{p,a}^2 \int_{-\infty}^{\infty} dp_\parallel p_\parallel \int_0^\infty dp_\perp \frac{p_\perp}{\gamma(\omega - kv_\parallel)} \frac{\partial F_a}{\partial p_\parallel}$$

$$= \frac{2\pi}{\omega} \sum_a m_a \omega_{p,a}^2 \underbrace{\int_{-\infty}^{\infty} dp_\parallel p_\parallel \int_0^\infty dp_\perp \frac{p_\perp}{\gamma(\omega - \frac{kcp_\parallel}{\sqrt{m_a^2 c^2 + p_\parallel^2 + p_\perp^2}})} \frac{\partial F_a}{\partial p_\parallel}}_{\text{involves complex logarithm}} \qquad (45)$$

To remove branch cut, transform to Lerche (1967) variables $y = p_\parallel/(m_a c)$, $E = \gamma = \sqrt{1 + ((p_\perp^2 + p_\parallel^2)/(m_a^2 c^2))}$ yielding with $z = (\omega_R + \imath\Gamma)/(kc) = R + \imath I$ for isotropic distributions $F_a(E)$

RUB

Collective eigenmodes (2)

$$\Lambda_L = 1 - \frac{4\pi}{k^2c^2}\sum_a (m_a c)^3 \omega_{p,a}^2 \int_1^\infty dE\, E\sqrt{E^2-1}\frac{\partial F_a}{\partial E}$$

$$+\frac{2\pi z}{k^2c^2}\sum_a (m_a c)^3 \omega_{p,a}^2 \int_1^\infty dE\, E^2 \frac{\partial F_a}{\partial E} J(E,z) \qquad (46)$$

with complex logarithm integral for $k > 0$ and $S(E) = \sqrt{1-E^{-2}}$

$$J(E,z) = J(E,R,I) = \int_{-S(E)}^{S(E)} \frac{dt}{z-t} = \int_{-S}^{S} \frac{dt}{R-t+\imath I}$$

$$= \int_{-S}^{S} dt\, \frac{R-t-\imath I}{(R-t)^2+I^2} = \frac{1}{2}\ln\frac{(S+R)^2+I^2}{(S-R)^2+I^2} - \imath \int_{-\frac{S+R}{I}}^{\frac{S-R}{I}} \frac{dx}{1+x^2}$$

$$= \frac{1}{2}\ln\frac{(S+R)^2+I^2}{(S-R)^2+I^2} - \imath\left[\arctan\frac{S(E)+R}{I} + \arctan\frac{S(E)-R}{I}\right] \qquad (47)$$

Complex logarithm integral

For $S(E) < R$ note that

$$\lim_{I\to 0^-}[\arctan\frac{S(E)+R}{I}+\arctan\frac{S(E)-R}{I}]$$

$$=\lim_{I\to 0^+}[\arctan\frac{S(E)+R}{I}+\arctan\frac{S(E)-R}{I}]=0 \tag{48}$$

However, for $S(E) \geq R$, corresponding to $E \geq (1-R^2)^{-1/2}$,

$$\lim_{I\to 0^+}[\arctan\frac{S(E)+R}{I}+\arctan\frac{S(E)-R}{I}]=\pi,$$

$$\lim_{I\to 0^-}[\arctan\frac{S(E)+R}{I}+\arctan\frac{S(E)-R}{I}]=-\pi, \tag{49}$$

so that the correct analytical continuation is

Complex logarithm integral (2)

$$J(E,R,I)=\frac{1}{2}\ln\frac{(S+R)^2+I^2}{(S-R)^2+I^2}-\imath[\arctan\frac{S(E)+R}{I}$$
$$+\arctan\frac{S(E)-R}{I}+\pi\sigma\Theta(1-R)\Theta(E-(1-R^2)^{-1/2})] \qquad (50)$$

with $\sigma=0$ for $\Gamma>0$ and $I>0$ and $\sigma=2$ for $\Gamma<0$ and $I<0$.
The last correct analytical continuation gives rise to the existence of damped longitudinal subluminal electrostatic waves and damped transverse aperiodic fluctuations in isotropic plasmas. Mathematical reason for Landau damping.

RUB

Thermal dispersion functions

For thermal distribution function $F_a(p) = C_a e^{-\mu_a E}$

$$\Lambda_L = 1 + \frac{\sum_a \omega_{p,a}^2 \mu_a}{k^2 c^2}[1 + L(\mu_a, z)],$$

$$L(\mu_a, z) = \frac{\mu_a z}{2K_2(\mu_a)} \int_1^\infty dE \, \frac{\partial u_2(E)}{\partial E} J(E, z),$$

$$u_2(E) = \frac{e^{-\mu_a E}}{\mu_a}\left(E^2 + \frac{2E}{\mu_a} + \frac{2}{\mu_a^2}\right), \quad E = \frac{1}{\sqrt{1 - S^2}} \tag{51}$$

Partial integration provides

$$L(\mu_a, z) = -\frac{\mu_a z}{2K_2(\mu_a)} \int_0^1 dS \, u_2(E) \frac{\partial J(E, z)}{\partial S} = -\frac{\mu_a z}{2K_2(\mu_a)} \int_{-1}^1 dS \, \frac{u_2(S)}{z - S} \tag{52}$$

For nonrelativistic temperatures $K_2(\mu_a) \simeq \sqrt{\pi/2\mu_a} e^{-\mu_a}$ and $u_2(E) \simeq E^2 e^{-\mu_a E}/\mu_a$, so that with the substitution $S = \tanh t$

RUB

Thermal dispersion functions (2)

$$L(\mu_a, z) \simeq -z\sqrt{\frac{\mu_a}{2\pi}} \int_{-1}^{1} \frac{dS}{(z-S)(1-S^2)}\, e^{-\mu_a\left(\frac{1}{\sqrt{1-S^2}}-1\right)}$$

$$= z\sqrt{\frac{\mu_a}{2\pi}} \int_{-\infty}^{\infty} dt\, \frac{e^{-\mu_a(\cosh t-1)}}{\tanh t - z} \simeq z\sqrt{\frac{\mu_a}{2\pi}} \int_{-\infty}^{\infty} dt\, \frac{e^{\frac{-\mu_a t^2}{2}}}{t-z}$$

$$= z\sqrt{\frac{\mu_a}{2}} Z\left(z\sqrt{\frac{\mu_a}{2}}\right) = \frac{z}{\beta_a} Z\left(\frac{z}{\beta_a}\right) \qquad (53)$$

with thermal velocity (in units of c) $\beta_a = \sqrt{2/\mu_a} = \sqrt{2k_B T_a/m_a}/c$ and the Fried-Conte plasma dispersion function

$$Z(x) = \pi^{-1/2} \int_{-\infty}^{\infty} dt\, \frac{e^{-t^2}}{t-x} \qquad (54)$$

Thermal nonrelativistic dispersion relations

Consequently (prime denotes differentiation of Z with respect to its argument),

$$\Lambda_L = 1 - \frac{\sum_a \omega_{p,a}^2}{k^2 c^2 \beta_a^2} Z' \left(\frac{z}{\beta_a} \right) = 0 \tag{55}$$

as $1 + xZ(x) = -2Z'(x)$.

Likewise, in magnetized isotropic plasmas for parallel ($\vec{k} \times \vec{B} = 0$) wave vectors (Gary 1993) with $b_a = \Omega_{0,a}/kc$ and $k = k_{\|}$

$$\Lambda_{\mathrm{RH,LH}}(k, z) = 1 - \frac{1}{z^2} + \sum_a \frac{\omega_{p,a}^2}{k^2 c^2 \beta_a z} Z\left(\frac{z \pm b_a}{\beta_a} \right) = 0 \tag{56}$$

Solution of the dispersion relation

$Z(x)$ and $Z^{'}(x)$ obey

$$Z(-x) = 2\pi^{1/2}\imath e^{-x^2} - Z(x), \quad Z^{'}(-x) = 4\pi^{1/2}\imath x e^{-x^2} + Z^{'}(x) \tag{57}$$

and have the asymptotic expansions

$$Z(x) \simeq \imath\pi^{1/2} e^{-x^2} - 2x\left[1 - \frac{2x^2}{3}\right], \; |x| \ll 1 \tag{58}$$

and

$$Z(x) \simeq \imath\sigma\pi^{1/2} e^{-x^2} - \frac{1}{x}\left[1 + \frac{1}{2x^2} + \frac{3}{4x^4}\right], \; |x| \gg 1 \tag{59}$$

where $\sigma = 0$ if $\Im(x) > 0$, $\sigma = 1$ if $\Im(x) = 0$, and $\sigma = 2$ if $\Im(x) < 0$, accounts for the analytical continuation.

RUB

Solution of the dispersion relation (2)

The dispersion relations read

$$\Re\Lambda(R,I)=0,\;\;\Im(R,I)=0 \tag{60}$$

The dispersion functions are complex analytical functions in the entire complex frequency plane i.e. they obey the Cauchy-Riemann relations

$$\frac{\partial\Re\Lambda(R,I)}{\partial R}=\frac{\partial\Im\Lambda(R,I)}{\partial I},\;\;\frac{\partial\Re\Lambda(R,I)}{\partial I}=-\frac{\partial\Im\Lambda(R,I)}{\partial R} \tag{61}$$

RUB

Weak damping/amplification limit

For $|I| \ll R$ Taylor-expand Eqs. (60) to 1st order around I=0:

$$0 \simeq \Re\Lambda(R, I=0) + I\left[\frac{\partial\Re\Lambda(R,I)}{\partial I}\right]_{I=0} = \Re\Lambda(R, I=0) - I\frac{\partial\Im\Lambda(R, I=0)}{\partial R},$$

$$0 \simeq \Im\Lambda(R, I=0) + I\left[\frac{\partial\Im\Lambda(R,I)}{\partial I}\right]_{I=0} = \Im\Lambda(R, I=0) + I\frac{\partial\Re\Lambda(R, I=0)}{\partial R} \tag{62}$$

To lowest order in the small quantity $(I/R)^2 = (\Gamma/\omega_R)^2 \ll 1$ then the dispersion relation of weakly damped/amplified fluctuations and the damping/growth rate are given by

$$\Re\Lambda(R, I=0) = 0, \quad I = -\frac{\Im\Lambda(R, I=0)}{\frac{\partial\Re\Lambda(R,I=0)}{\partial R}} \tag{63}$$

For consistency, the resulting weak damping solutions have to fulfil $|I| \ll R$, which has to be checked aposteriori.

Weak propagation limit

Likewise in the weak propagation limit $|I| \gg R$ Taylor-expand Eqs. (60) to 1st order around R=0:

$$0 \simeq \Re\Lambda(R=0,I)+R\left[\frac{\partial\Re\Lambda(R,I)}{\partial R}\right]_{R=0} = \Re\Lambda(R=0,I)+R\frac{\partial\Im\Lambda(R=0,I)}{\partial I},$$
$$0 \simeq \Im\Lambda(R=0,I)+R\left[\frac{\partial\Im\Lambda(R,I)}{\partial R}\right]_{R=0} = \Im\Lambda(R=0,I)-R\frac{\partial\Re\Lambda(R=0,I)}{\partial I} \quad (64)$$

To lowest order in the small quantity $(R/I)^2 = (\omega_R/\Gamma)^2 \ll 1$ then the dispersion relation of weakly propagating fluctuations and the corresponding real parts are given by

$$\Re\Lambda(R=0,I)=0, \quad R=\frac{\Im\Lambda(R=0,I)}{\frac{\partial\Re\Lambda(R=0,I)}{\partial I}} \quad (65)$$

Again, check aposterior that $R \ll |I|$.

RUB

Weak damping slab MHD eigenmodes

According to Eqs. (56) and (63) the dispersion relation of weak damping slab transverse waves is given by

$$0 = \Re\Lambda_{\mathrm{RH,LH}}(k,R) = 1 - \frac{1}{R^2} + \sum_a \frac{\omega_{p,a}^2}{k^2c^2\beta_a R}\Re Z\left(\frac{R \pm b_a}{\beta_a}\right) \qquad (66)$$

With property (56) that $\Re Z(-x) = -\Re Z(x)$ we readily obtain $\Re\Lambda_{\mathrm{LH}}(k,-R) = \Re\Lambda_{\mathrm{RH}}(k,R)$. Hence, instead of analyzing the two dispersion relations for RH and LH polarized waves for $k > 0$ and $R > 0$, it suffices to analyze one of these (LH is standard choice) for $k > 0$, but allow positive (for LH) and negative (for RH) values of R and ω_R.

In equal thermal temperature ($T_e = T_p$) electron-proton plasma with the mass ratio $\chi = m_e/m_p = 1/1836$ one finds $b_p kc = \Omega_{0,p} = qB_0/m_p c$ $= -\chi\Omega_{0,e} = -\chi b_e kc$, $\omega_{p,p}^2 = \chi\omega_{p,e}^2$ and $\beta_p = \chi^{1/2}\beta_e$ so that the LH-dispersion relation is given by

RUB

Weak damping slab MHD eigenmodes (2)

$$0 = \Re\Lambda_{\rm LH}(k>0,R) = 1 - \frac{1}{R^2} + \frac{\omega_{p,e}^2}{k^2c^2\beta_e R}\left[\Re Z\left(\frac{R-b_e}{\beta_e}\right) + \chi^{1/2}\Re Z\left(\frac{R-b_p}{\chi^{1/2}\beta_e}\right)\right], \qquad (67)$$

which has been analyzed by RS and Skoda (2010, ApJ 716, 1596).

In the cold plasma limit $\beta_e \to 0$ the arguments of the Z-function are large ($|x| \gg 1$), so that we use the expansion (59) that $\Re Z(x) \simeq -x^{-1}$ yielding

$$0 = 1 - \frac{1}{R^2} - \frac{\omega_{p,e}^2}{k^2c^2R}\left[\frac{1}{R-b_e} + \frac{\chi}{R-b_p}\right] \qquad (68)$$

Cold plasma limit

For large $|R| \gg |b_e| \gg b_p$ we find as solution of the disperrsion relation

$$R^2 \simeq 1 + \frac{\omega_{p,e}^2(1+\chi)}{k^2c^2}, \text{ corresponding to } \omega_R^2 = \omega_{p,e}^2(1+\chi) + k^2c^2, \quad (69)$$

which are superluminal electromagnetic waves.

For subluminal values $|R| \ll 1$ we obtain the cold plasma dispersion relation

$$k^2c^2 \simeq \omega_{p,e}^2 [\frac{\chi R}{b_p - R} - \frac{R}{R - b_e}] = \omega_{p,e}^2 [\frac{\chi \omega_R}{\Omega_{0,p} - \omega_R} - \frac{\omega_R}{\omega_R + |\Omega_{0,e}|}] \quad (70)$$

Its solutions, shown in Fig. 3, are the LH-polarized Alfven-ion-cyclotron (AIC) branch and the RH polarized magnetosonic-Whistler (MSW) branch.

Cold plasma limit (2)

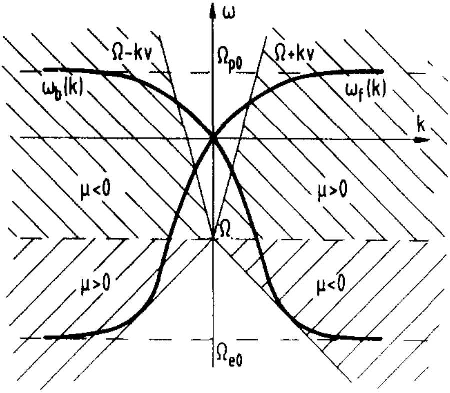

Figure 3: Subluminal slab MHD eigenmodes in a cold electron-proton plasma.

Cold plasma limit (3)

For frequencies $|\omega_R| \ll |\Omega_{0,e}|$ the subluminal dispersion relation (70) reduces to (with $\Omega_p = \Omega_{0,p}$)

$$\frac{\omega_R^2}{k^2 V_A^2} \simeq \frac{\Omega_p - \omega_R}{\Omega_p} \tag{71}$$

with the Alfven speed

$$V_A = \frac{c\Omega_p}{\omega_{p,p}\sqrt{1+\chi}} = \frac{B_0}{\sqrt{4\pi(m_e + m_p)n_e}} = 2.18 \cdot 10^6 \frac{(B_0/\mu\mathrm{G})}{(n_e/1\,\mathrm{cm}^{-3})^{1/2}} \ \frac{\mathrm{cm}}{\mathrm{s}} \tag{72}$$

much smaller than c. The two solutions of Eq. (71) are the LH-circularly polarized AIC-branch

Cold plasma limit (4)

$$\omega_R = kV_A[\sqrt{1+(\frac{k}{k_c})^2} - \frac{k}{k_c}] \simeq \begin{cases} kV_A & \text{for } |k| \ll k_c \text{ (Alfven wave)} \\ \Omega_p[1-(\frac{k_c}{2k})^2] & \text{for } |k| \gg k_c \text{ (Ion cyclotron wave)} \end{cases} \quad (73)$$

and the RH-circularly polarized MSW-branch

$$\omega_R = -kV_A[\sqrt{1+(\frac{k}{k_c})^2} + \frac{k}{k_c}] \simeq \begin{cases} -kV_A & \text{for } |k| \ll k_c \text{ (Alfven wave)} \\ -\frac{V_A^2 k^2}{\Omega_p} & \text{for } |k| \gg k_c \text{ (Whistler wave)} \end{cases} \quad (74)$$

with the characteristic wavenumber

$$k_c = \frac{2\Omega_p}{V_A} = 2\omega_{p.e}\chi^{1/2}/c = 8.78 \cdot 10^{-8}(n_e/1\ \mathrm{cm}^{-3})^{1/2}\ \ \mathrm{cm}^{-1} \quad (75)$$

Cold plasma limit (5)

Favorite agreement with general calculations.

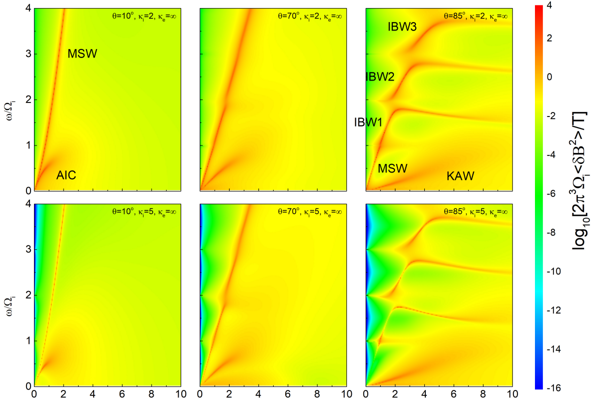

Figure 4: Magnetic fluctuation spectra from theory and simulations (Kim et al. 2018)

RUB

Cold plasma limit (5)

Induction law (40) for Alfven waves,

$$< B_\perp^2 > (k, \omega_R) = \frac{c^2 k^2}{\omega_R^2} < E_\perp^2 > (k, \omega_R)$$

$$= \frac{c^2}{V_A^2} < E_\perp^2 > (k, \omega_R) \gg < E_\perp^2 > (k, \omega_R), \quad (76)$$

indicates that $|\delta B| \gg |\delta E|$. Alfven waves have much larger magnetic than electric fluctuations!

Summary and conclusions

- The kinetic theory of plasma fluctuations has been developed with no restriction on the frequency ω of the fluctuations.
- For any isotropic particle distribution function the spectral electromagnetic fluctuation spectra resulting from spontaneous emission can be calculated analytically for unmagnetized and magnetized plasmas. The spectra are dominated by collective eigenmodes.
- Tremendous progress over the last 5 years with the analysis of fluctuations in magnetized plasmas.
- Aperiodic transverse fluctuations are the fourth collective eigenmode in unmagnetized plasmas. In the early universe the stochastic electric and magnetic fields have tera-Gauss strength. These stochastic magnetic fields also serve well as cosmological seed fields for further amplification processes once additional kinetic free energy becomes available in these plasmas (a new primary dynamo process).